Annotation

The main topic of this thesis is the post-crisis pricing of OTC derivatives. There has been a shift in derivatives valuation since the 2007-2008 crisis, which had significant impact on the financial markets. The regulatory environment has changed along with accounting rules and the market practitioners started to include the costs and benefits of derivatives trading to the value of the derivative itself. Credit value adjustment (CVA) and debt value adjustment (DVA) were already accounted for before the crisis, but the gained larger momentum afterwards. The shift in valuation brought further value adjustments into life, which are collectively called xVAs and include funding value adjustment (FVA), capital value adjustment (KVA), margin value adjustment (MVA) etc.

In general derivatives pricing is a dynamically changing field which is impacted by the regulators, accounting rules and the market practitioners, as well. It is interesting how a plain vanilla interest rate swap trade can result in large-scale simulations, if one wants to consider every factor e.g. funding, capital requirements, collateralization for the whole lifetime of the trade. This requires a number of assumptions and approximation as it is not possible for example to predict the regulatory regime for twenty years in advance. Also besides that, one must consider the behavior of the counterparty, as well. Therefore from a quite simple swap pricing exercise one ends up simulating a great number of risk factors and implementing several numerical methods.

The xVA terms can be seen as complex exotic derivatives where the underlying is the relevant portfolio of the bank. Therefore xVAs can be risk managed and need to be integrated into front-office decision making systems centrally so that the bank can optimize their trading strategy and remain profitable even with rising capital requirements and tightening margins.

The main purpose of this thesis is to provide a comprehensive view of xVA terms and compare the value of the same netting set - consisting of two interest rate swaps - with three different counterparts having different margin agreement and counter-
party risk. When modeling xVA terms one must try to find an optimal solution, which is still useful, but not extremely complex and can be implemented without extensive IT infrastructure costs. This thesis assumes several approximations, which makes the implementation in R sustainable.

In the first part of the thesis, the research problem, motivation and objectives were stated along with an introduction to the post-crisis derivatives markets. The second part provided a high level overview of derivatives markets and practitioners focusing on investment banks. In terms of derivatives valuation it presented the traditional risk-neutral pricing framework in contrast with the recent so-called Value-To-Me concept including xVAs. The third part focused on introducing several risk factors and the relating xVA components. It highlighted several considerations about xVA in general. The fourth part presented step-by-step a simple xVA implementation and provided a primer on the numerical results.

The optimal total xVA charge depends on counterparty credit risk, funding costs, capital requirements, and many other factors. Hence, the choice of hedge and collateral might differ depending on these issues. This thesis concludes that the value of a netting set can be different depending on the counterparty, the CSA agreement and other specifications and therefore confirms the need for xVA calculation. What is more, in some cases xVA amounts can be quite significant compared to the risk neutral price therefore they need to be carefully considered and accounted for.

However popular research topic it is, the xVA field itself is quite recent, therefore there are still debates around certain xVA terms and how to model them. In my view it will take some time to have clear and concise terminology along with best practices from the market. I would argue that the most challenging problem in the xVA world is to set up the modeling infrastructure, which is robust enough without being too costly to integrate into decision making systems. A proper xVA modeling framework should be able to achieve estimates at a high level of precision without significantly increasing the computation time. However regarding investment banks, another great challenge in terms of adopting xVA will be to modify and adjust processes, organizational structures and control functions.

The field of xVA can be considered for an interdisciplinary research area as experience in computer science and IT infrastructure would be helpful. On the other hand it can be interesting from management, human resources, process engineering and accounting perspective, as well.

All in all, a trend of moving towards standardized vanilla trades can be identified, given exotic derivatives are getting more and more costly to trade, but the landscape of derivatives markets is continuously changing due to regulatory pressure,
accounting rules and unexpected events like the 2007-2008 crisis. In my opinion the growing xVA desks in banks and central counterparty clearing houses will have essential role in the derivatives trading business.
Modern pricing of OTC derivatives
xVAs involved

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Firstly, I would like to thank my thesis supervisor, Balázs Márkus for his encouragement and constructive ideas for the thesis. Also I would like to express my gratitude to Bence Szöllős for his feedback and the relevant discussions, which have inspired my choice of thesis topic.
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Part I:

Formalia

This part focuses on the introduction to the main topics of this thesis. It states the motivations and the research methodology for the given objectives.
1 Introduction

The main topic of this thesis is the modern pricing framework of OTC derivatives. There has been a shift in derivatives valuation since the 2007-2008 crisis, which had significant impact on the financial markets. The regulatory environment has changed along with accounting rules and the market practitioners started to include the costs and benefits of derivatives trading to the value of the derivative itself. Credit value adjustment (CVA) and debt value adjustment (DVA) were already accounted for before the crisis, but the gained larger momentum afterwards. The shift in valuation brought further value adjustments into life, which are collectively called xVAs and include funding value adjustment (FVA), capital value adjustment (KVA), margin value adjustment (MVA) etc.

There are several ways to group and name these xVAs, and even more ways to model them. What is common in most approaches is that they rely heavily on large-scale simulations of exposure profiles and xVA integrals. In order to make computations more efficient, approximations or analytic solutions can be applied in some cases.

The xVA terms can be seen as complex exotic derivatives where the underlying is the relevant portfolio of the bank. Therefore xVAs can be risk managed and need to be integrated into front-office decision making systems centrally so that the bank can optimize their trading strategy and remain profitable even with rising capital requirements and tightening margins.

The main purpose of this thesis is to provide a comprehensive view of xVA terms and compare the value of the same netting set - consisting of two interest rate swaps - with three different counterparts having different margin agreement and counterparty risk.

When modeling xVA terms one must try to find an optimal solution, which is still useful, but not extremely complex and can be implemented without extensive IT infrastructure costs. This thesis assumes several approximations, which makes the implementation in $R$ sustainable.

1.1 Motivation

The changing landscape of derivatives pricing has caught my attention during my internship at an investment bank. I was in a team responsible for validating interest rate derivatives valuation and risk models of the bank. This practical experience highlighted for me that the theory can sometimes differ from real markets.

In general derivatives pricing is a very dynamically changing field which is im-
pacted by the regulators, accounting rules and the market practitioners, as well. There are some highly structured products on the market, which are close to impossible to model. However it is interesting how a plain vanilla interest rate swap trade can result in large-scale simulations, if one wants to consider every factor e.g. funding, capital requirements, collateralization etc. for the whole lifetime of the trade. This requires a number of assumptions and approximation as it is not possible for example to predict the regulatory regime for twenty years in advance. Also besides that, one must consider the behavior of the counterparty, as well. Therefore from a quite simple swap pricing exercise one ends up simulating a great number of risk factors and implementing several numerical methods.

1.2 Thesis structure

The structure of the thesis is organized in a way that it starts from giving an overview of the recent shift in derivatives valuation, then it outlines the xVA terms and shows the steps of the xVA calculation through a simple implementation.

Part II provides basic introduction to derivatives markets and its participants. It also covers the recent changes in the regulatory environment of the financial sector. It shows derivatives from an investment bank’s perspective and outlines the life-cycle and profitability of derivatives transactions. It also briefly covers fair value accounting principles. Valuation of derivatives is also included, covering the traditional risk neutral pricing framework and introducing the so-called Value-To-Me concept.

Part III first mentions several significant considerations about xVA calculation in general. Then it gives detailed introduction to the majority of the different risk types with respect to OTC derivatives that are covered by the xVA components. This part gives a comprehensive overview of xVA components, which are implemented in the next part.

Part IV highlights different assumptions of implementing xVA calculations and it also gives a summary of the numerical results along with its limitations.

Part V concludes the findings and observations of the thesis and proposes potential topics for further research.
PART II:

NEW ERA IN DERIVATIVES MARKETS

This part gives a basic introduction to derivatives markets and its participants. It also covers the recent changes in the regulatory environment of the financial sector. It looks at derivatives from an investment bank’s perspective and shows the life-cycle, profitability of derivatives transactions and also briefly covers fair value accounting principles. Valuation of derivatives is also included here, covering the traditional risk neutral pricing framework and introducing the so-called Value-To-Me concept.
1 Derivatives overview

This section gives a high level overview of derivatives markets including trading mechanism, market participants and the impacts of the recent crisis on derivatives pricing along with the improvements of the regulatory ecosystem.

1.1 Market overview

As financial derivatives have been actively traded for many decades now, derivatives markets have become rather complex. The main reason of their increasing volume is that they transfer risk from one party to another in a tailor-made way with low transaction costs compared to investing directly to the underlying. They also provide hedging and risk protection and require minimal upfront investment due to leverage. The different expectations on future market movements and risk appetite of market participants resulted in a wide variety of structured products. (Deutsche Börse Group, 2008)

Derivatives are useful and contribute to economic growth, therefore can be considered socially useful, especially in cases where they reduce risk. However, they can magnify risks, as well and cause financial losses or even crises. This is due to the fact that they carry material risk themselves. Derivatives do not necessarily eliminate financial risk, but they convert it into other forms. A good example to demonstrate this, is using collateral to reduce counterparty risk, while creating market, operational and legal risk at the same time. Derivatives also carry credit and liquidity risk. Nevertheless, due the the highly concentrated over-the-counter (OTC) market, systemic risk is now one of the biggest risk factors in the current market environment. (Gregory, 2015)

In general, derivatives can be exchange traded or OTC. The standardized exchange traded derivatives carry less operational, credit, counterparty risk, but they are not flexible enough.

To enhance transparency in derivatives transactions and mitigate counterparty risk - and to some extent systemic risk- the OTC market has moved from non-collateralized bilateral contracts towards more transparent and collateralized transactions. Vanilla derivatives are mostly traded through clearing houses. However, there are some tailor-made and bespoke structured derivatives transactions that cannot be off-loaded to central counterparty clearing houses (CCP) and there is no guarantee that even all standardized derivatives can be cleared. Whether or not the transaction is suitable for clearing, is contingent on rather the standardization of operational and legal aspects than its economic features. The non-cleared derivatives
play a key role in many sectors and are extensively used by different entities, e.g. pension funds, corporations, financial institutions. As a consequence, they cannot be fully replaced with cleared derivative trades. (ISDA, 2013)

The participants of the OTC market can be divided into derivative providers and derivatives users. The market is mainly driven by the expert derivative providers - large players known as dealers. They have the expertise in trading, valuation and risk management of derivatives. They are often market makers, providing two-way prices and liquidity to the market. Dealers are mostly large global banks, investment banks and other financial institutions with exposure to many clients and counterparties, with derivatives in their trading books across all asset classes. Derivative users - known as customers - can be corporations, financial institutions such as pension funds, hedge funds, insurance companies and smaller banks. They have only few derivative positions and often in a single asset class with exposure to smaller number of counterparties. They are most likely to enter into derivative positions for hedging or yield enhancement. (Lu, 2015)

Derivatives trading is mostly driven by supply and demand on the so-called broker market, the market between dealers linked by brokers. Although it can be pretty concentrated for less frequently traded products, most dealers are willing to participate in any market activities that other dealers do. The reason for that is that the inquiries and quotes from other dealers is essential to have a good understanding of the market. Dealers also hedge their risks from trading with clients using the broker market. (Lu, 2015)

The trading between dealers and customers takes place on the so-called customer market. The market is driven by auction mechanism. The customer asks several dealers for the same trade and will enter into the most beneficial agreement. This results in a competition among dealers for the same contract and can result in less profit for the winner of the auction. This phenomenon is called "winner’s curse". The winner of the auction tends to overpay due to incomplete information or mispricing. (Lu, 2015)

According to Bank for International Settlements (BIS) derivatives end-users can be either other financial institutions or non-financial customers as summarized in Table 1.

1.2 Impacts of the recent crisis

The global financial crisis of 2007-08 (referred to as the crisis) has left serious marks on the financial sector from many aspects - a new era has started both in terms of valuation and regulation of derivatives.
Table 1: Categorization of derivatives (Table edited by author based on (BIS, 2016))

<table>
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<tr>
<th>Derivatives providers</th>
<th>Derivatives users</th>
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<tbody>
<tr>
<td><strong>Reporting dealers</strong></td>
<td>Other financial institutions</td>
</tr>
<tr>
<td>investment banks</td>
<td>pension funds</td>
</tr>
<tr>
<td>large banks</td>
<td>small regional banks</td>
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<td></td>
<td>hedge funds</td>
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The change in discounting is a key factor from a valuation perspective. Before the crisis, the London Inter bank Offered Rate (LIBOR) rates were used as a proxy for the risk-free rate as banks with AA ratings were considered default-free. This view has changed with the near-default of Bear Stearns and Lehman’s default. This showed that LIBOR rates do carry credit risk, as they contain risk premium over collateralized lending rates between banks. After the crisis the Overnight Index Swap (OIS) rate has been used as risk-free rate. It could be a better approximation of the risk-free rate as OIS rate is based on actual trades (not like LIBOR fixings). OIS rate is the fix rate that has to be paid in exchange for the daily overnight rate, which is the interest rate on cash collateral, therefore the OIS rate has minimal intrinsic credit risk. There is a liquid OIS market for several maturities and currencies. (Lichters et al., 2015)

After the crisis it was clear to the financial sector that it should be one of the major goals, to address counterparty and systemic risk appropriately. One initiative to achieve this was establishing trade repositories to whom all derivatives transactions need to be reported. Another way of mitigating counterparty risk is moving towards clearing of OTC derivatives through clearing houses. (ISDA, 2013)

The major difference between bilateral and cleared derivatives trading is that in latter case the parties trade with a central counterpart (the CCP), whereas bilateral trading means counterparty risk exposure for each contract as parties trade with each other directly as seen in Figure 1.

In the recent years clearing houses have stepped up to a key position on the OTC market. Their role is getting more and more significant as market players are highly encouraged by regulators to conduct their trading through CCPs. Clearing houses can have direct members who contribute to a guarantee fund, which protects the members and the CCP against default.
These direct members can have clients that also want to clear with the CCP. In case a direct member defaults, the guarantee fund needs to be recollected. As a consequence each member has counterparty risk exposure to all other parties and the CCP itself. The counterparty risk exposure of the members is essentially driven by two factors. On one hand each member is exposed to the current value of their portfolio with the other party and also the potential losses that they would face in case of the counterparty’s default. Variation Margin is posted to mitigate the current value risk factor and Initial Margin mitigates the risk arising from future losses due to adverse moves of the portfolio. Variation margin is re-balanced on a regular basis and accrue interest on the overnight rate. Central clearing however does not fully mitigate counterparty risk, but rather transforms it into liquidity risk due to margin requirements. (Lichters et al., 2015)

Nowadays due to regulatory pressure the market is moving rapidly towards clearing OTC derivatives through CCPs. According to International Swaps and Derivatives Association (ISDA) this trend can hold up until 70 percent of global OTC derivatives trading is cleared. (ISDA, 2013)

The amount of cleared is higher compared to non-cleared interest rates (IR) derivatives. Currently roughly the half of interest rate swaps (IRS) are cleared along with floating rate agreements (FRA), OIS and basis swaps. Cross-currency swaps, swaptions and caps and floors are not yet fully eligible for central clearing. (Domanski et al., 2015)

The majority of non-cleared derivatives transactions are collateralized to address counterparty risk. The collateral agreement is stated in the Credit Support Annex (CSA). The CSA is a collateral provision of the ISDA master agreement, which is a contractual framework between counterparties of the OTC market and allocates
the financial risks between parties without stating any trade specific details. Transactions under CSA are netted, the party in the out-of-the-money (OTM) position posts collateral to the other party. The portfolio and the collateral is revalued (and if needed the collateral is adjusted) regularly. Collateralization is a key factor also from valuation perspective as unsecured cash instruments are mixed with collateralized derivatives. (Lichters et al., 2015)

One of the major changes after the crisis is the regulatory environment for derivatives trading. The regulatory reform has started with the implementation of the major overhaul of Basel II. in 2011 - became known as Basel II. 5. It was clear that the changes proposed by the Basel Committee on Banking Supervision (BCBS) were necessary. The proposal included introducing a new incremental risk charge (IRC) to re-balance the chaotic requirement differences between the trading book and the banking book. Besides that, the calculation of stressed Value at Risk (VaR) and a comprehensive risk measure (CRM) for instruments contingent on credit correlation of different assets were also proposed. (Hull, 2015)

The final version of Basel III. - the current regulation - was published in 2010 and is being implemented gradually between 2013 and 2019. Among other things, more focus is put on liquidity and counterparty credit risk and it also provides more clarity around capital definition and requirements. In the US, The US DoddFrank Wall Street Reform and Consumer Protection Act 2009 (DoddFrank) was implemented to provide end-users protection, enhance transparency and prevent possible future bailouts of financial institutions. (Hull, 2015)

The European implementation of post-crisis regulations are referred to as Capital requirements regulation and directive (CRR/CRD) proposed by the European Commission. European Market Infrastructure Regulation (EMIR) - the regulation on OTC derivatives, central counterparties and trade repositories - was also adopted and entered into force for the sake of transparency, stability and efficiency around OTC markets. (Svoboda and Reuse, 2012)

These regulations established a sound control framework for financial markets in both Europe and the US. As a result of the increased regulatory capital requirements derivatives can become more costly therefore valuation should reflect all charges associated with trading.

## 2 From an investment banks’ perspective

Typically investment banks can be divided into three main areas, such as front office, middle office and back office. Front office interacts with clients, conducts
business and research and it generates the revenue for the bank. The back office has a supporting function - it creates the infrastructure and provides analytic solutions for the front office and it also creates value for the shareholders via controlling the activity of the front office from an operational, legal and technical point of view. The mid-office is somewhere between the these two and it can be varying in different institutions. The purpose of the mid-office in general is to manage and control the risks taken by front office and also to help funding their business activities and to provide additional financial control. The three areas are highly separated to ensure segregation of duty, but also should collaborate to ensure a coherent firm-wide strategy.

The derivatives trading activity is conducted by the front office, which is typically divided into business units based on asset class, which can be further divided into desks depending on product types they are dealing with. The trading desks trade with clients, but also do back-to-back trades with other desks in the same bank, as well. The idea is that a desk should only carry the risk they can hedge. For instance if a structured rate desk issues a note with equity contingent coupon, it would do an internal trade with the equity desk to transfer the risks arising from the equity component. This way each desk would only deal with risks the trader can manage on the market.

2.1 Trade life-cycle and profitability

Trading derivatives involves wide range of departments e.g. sales and trading, operations, legal, risk, finance, quantitative modelers, technologist and so forth, depending on the complexity and size of the transaction. The trading process can be divided into pre-trade, trade and post-trade parts as seen in Table 2.

The whole trading process begins with customer-sales negotiation. The more bespoke the customer’s requirement is, the more time it takes to create tailor-made products. After looking at the market and the risks involved, traders come up with a price including hedging costs. In cases of new counterparts or structured products, legal work also needs to be done. Risk management has high importance when it comes to risk limits, especially with significant trades that can hugely affect the risk exposure or increase the credit risk concentration. These limits are mostly for market and counterparty credit risk. The quantitative modelers are also key in the process as they develop the models to value derivatives. Also financial controllers can be involved to assess fair value compliance. (Lu, 2015)
After effective date, the derivative contract needs to be serviced until expiration or termination. Trades needs to be risk managed by the trader and marked correctly by financial control groups. Market data accuracy, model consistency, model and liquidity reserves are also key aspects.

In the new era of derivatives trading, the focus is on sustainable trade profitability. With shrinking margins - as seen in Figure 2 - and increasing capital and regulatory requirements in the derivatives trading business, it has become crucial to fully understand trade life-cycle and manage trade profitability.

It is critical to adopt a more holistic approach when assessing trade profitability even in pre-trade analysis. Introducing a trade Economic Value Added (EVA) framework would allow banks to quantify all of the costs related to a given trade and evaluate profitability on trade level to support pre-trade decision making. The aim is to structure trades that are profitable even when all costs are incorporated. Applying a holistic approach, one can look at derivatives trading as an optimization problem between different functions of a given bank as seen on Figure 3. (Kancharla, 2014)

The goal, is to increase profitability, which would mean restructuring current processes and implementing an integrated trading and risk system including intra-day
Table 2: Trading process of derivatives (Table edited by author based on (Lu, 2015, p.13))

<table>
<thead>
<tr>
<th>Pre-trading</th>
<th>Trading</th>
<th>Post-trading</th>
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<tbody>
<tr>
<td>Understand user requirements</td>
<td>Auction process</td>
<td>Reset and payment</td>
</tr>
<tr>
<td>Develop product</td>
<td>Booking transaction</td>
<td>Exercise and notification</td>
</tr>
<tr>
<td>Model/infrastructure</td>
<td>Marking value</td>
<td>Margin call and collateral exchange</td>
</tr>
<tr>
<td>configuration</td>
<td>Model, liquidity and other reserve</td>
<td>Valuation disputes</td>
</tr>
<tr>
<td>Risk assessment and approval</td>
<td></td>
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</tr>
</tbody>
</table>

or real-time pre-trade analysis. This can be challenging from IT infrastructure and process engineering perspective, as well. Therefore an efficient risk, collateral and capital allocation system integrated into the front office would create competitive edge in the derivatives business. (Kancharla, 2014)

Investment banks have had CVA desks even before the crisis, but these are getting broader focus and gaining more attention as xVA terms are getting recognized in financial markets. These xVA desks will have a key role in xVA optimization and management including pricing, hedging, and allocation of prices or hedging costs to trading desks on an individual trade basis. The interaction of xVA desk in the organization is illustrated in Figure refxvadesks. (Kenyon and Green, 2014)

2.2 Marking-to-market

Banks are required to classify their instruments either held-for-sale - these are in the trading book and have to be marked to market daily - or held-to-maturity, which are in the banking book and their values are not changed unless they become impaired.

There is an important distinction between the trading book and the banking book. The trading book contains all assets and liabilities associated with the trading activity, whereas banking book typically includes loans. The value of the positions in the trading book is marked-to-market or marked-to-model to reflect market changes in the books and records. As the banking book’s positions are mainly loans, they are not marked-to-market, but categorized by performance. Frequent revaluation
of the trading book’s positions using models calibrated to the market is referred by accountants as fair value hedge accounting. It involves an estimation for each instrument in the investment bank’s portfolio and also calculating the total value of the portfolio. It is a key component to determine capital requirements, as well. (Hull, 2015)

Calculating the mark-to-market or fair value of a position depends on the type of the transaction and also on available independent and reliable external data. Typically the positions are marked to the mid price, but in some cases the bank can chose to mark a long position to the bid and a short position to the offer.

Usually there is a pricing hierarchy on external data. The most favorable would be to see exchange traded quotes for all positions as it would then be pretty straightforward to mark them to market. The intra-dealer broker quotes are also good source, such as ICAP, Tullet Prebon, as financial institutions often trade through brokers so they have relevant and up-to-date information. Swap execution facilities (SEF) can provide information on more standardized OTC trades. For more bespoke trades consensus price can be obtained from Markit Group, which ensures that the bank marks their positions consistently with the other market participants. To obtain quotes from Markit each bank needs to submit their own quotes for a number of test trades and after eliminating outliers -who will not get back consensus quotes - an average quote is calculated and sent back to the participating banks. In case of exotic and structured products, only the pricing inputs can be sourced from external data. These trades are marked-to-model as the inputs feed into valuation model,
which generates the fair value. (Hull, 2015)

From accounting point of view, determining the fair value of derivatives transactions is not always straightforward. Accounting rules are somewhat standardized, but can vary country by country, which makes accounting of derivatives even more complex. The basic concept for derivatives is Fair Value Hedge Accounting (FVHA). FVHA allows banks to recognize profit and loss, as well as revenues and expenses in the same accounting period that come from a hedged item and a hedging instrument. The rules are described in the International Accounting Standards (IAS 39) for the EU and others and the Financial Accounting Standards Board (FAS 133) for the US. IAS 39 will be replaced by the International Financial Reporting Standard (IFRS) 9 on 1 January 2018. (Lichters et al., 2015)

3 Derivatives valuation

This section first focuses on the traditional risk-neutral pricing approach, then introduces the Value-To-Me concept and the xVA terms, which will be discussed in the next part of this thesis.

3.1 Risk neutral pricing framework

The problem of pricing financial assets boils down to the basic idea that the fair price of an asset should equal to the risk neutral expectation of the discounted future cashflows. It seems pretty intuitive and simple, but a few questions may arise. What
should be the discount factor? How can one define the probabilities to calculate the expected value of cash-flows? How can contingent claims (i.e. derivatives) be priced? To seek answer to these one needs to have a closer look on financial mathematics toolkit.

In financial mathematics the markets are constructed in a way that the findings of the stochastic calculus remain valid. These market models can be for single or multiple assets, complete or incomplete markets with discrete or continuous time horizon.

A key connection between financial markets and stochastic calculus is the Fundamental Theory of Asset Pricing, which is key to risk neutral pricing of multiple assets driven by multiple Brownian motions. This is a tailor-made application of Girsanov’s and Martingale Representation Theorem to the financial markets. (Shreve, 2004)

Girsanov’s theorem states that it is possible to change to an equivalent probability space so that the Brownian motion can be adjusted accordingly to still remain one. The application of this theorem is the change of measure of a financial market, i.e. using risk neutral martingale measure for pricing contingent claims. The Martingale Representation Theorem states that Brownian local martingales can be represented as stochastic integrals with respect to Brownian motion. The application to financial markets is that all contingent claims can be replicated by a dynamic trading strategy of the underlying. (Bingham and Kiesel, 2004)

The argument of no arbitrage pricing lies on the existence of an equivalent probability martingale measure on the underlying probability space under which the price process of the underlying becomes a martingale. This way one can change the real-world probability measure and add more weight to unfavorable events and less to more favorable ones as a result of risk aversion. (Delbaen and Schachermayer, 1994)

An attainable contingent claim’s payoff can be reproduced with a portfolio of underlying in a way that the discounted value of the contingent claim matches the costs and gains of setting up and trading the replicating portfolio. Supposing that the law of one price holds, one can argue that the no-arbitrage condition implies that the fair price of an attainable contingent claim equals the value of the replicating strategy. An admissible self-financing strategy gives a lower boundary to losses on short positions to ensure that if the wealth of the investor goes below zero they would still be able to pay off their debt. It can be proven that any attainable contingent claim can be uniquely replicated and the arbitrage-free price of this claim will be given by the value of any replicating strategy.
In a complete market all contingent claims can be replicated. The already mentioned Fundamental Theorem of Asset Pricing requires that contingent claims such as options can be perfectly hedged using the replication portfolio of underlying assets. This theorem also states that there exists a unique martingale equivalent measure on an arbitrage-free complete market. This can be derived from the combination of the no arbitrage theorem and the completeness theorem. The former states that the equivalent martingale measure exists and the latter argues that it is unique if all contingent claims can be replicated, i.e. the market is complete. (Bingham and Kiesel, 2004)

In discrete time models the change to risk neutral measure does not mean that the binomial tree itself changes, but the probabilities of each branch. However in continuous time the mean rate of return changes when switching to risk neutral measure, but the volatility of the underlying remains the same. The volatility determines the possible underlying price paths and with change of measure only the probabilities associated with the paths are changed. In risk neutral measure the probability on a path with lower return, is higher. (Shreve, 2004)

There are three scenarios for building models on multi-asset markets as shown in Table 3. The first scenario is not arbitrage free and therefore cannot be used for pricing. The second is not arbitrage-free either as it can give different prices for the same derivative based on the risk neutral measure, but the model can be calibrated to market prices and used for pricing non-traded instruments. The third case is arbitrage-free and the risk-neutral pricing formula can be used. (Shreve, 2004)

Table 3: Scenarios of multiasset market models (Table edited by author based on (Shreve, 2004))

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bad Model</strong></td>
<td><strong>Incomplete model</strong></td>
<td><strong>Complete Model</strong></td>
</tr>
<tr>
<td>No risk neutral measure exists</td>
<td>Multiple risk neutral measures exits</td>
<td>Unique risk neutral measure</td>
</tr>
<tr>
<td>Arbitrage opportunity</td>
<td>Different measures lead to different prices, but still can be used</td>
<td>Arbitrage-free pricing formula</td>
</tr>
</tbody>
</table>

In Cochrane’s view there are three ways to look at arbitrage pricing theory: martingale representation, state price representation and risk neutral pricing. The bundling of contingent claims can be given as $p = E(mx)$. On complete markets the
the stochastic $m$ discount factor is the set of contingent claim prices weighted by probabilities. As a result the discount factor and probability combination is referred to as state-price density. The discount factors can be interpreted as transformation to risk-neutral measure. The law of one price states that a discount factor exists and the no-arbitrage argument states that this discount factor is positive, however it does not state that it is unique. (Cochrane, 2005)

### 3.2 Value-To-Me concept

A vanilla interest rate swap was priced with a single yield curve model up until the recent years. Nowadays a valuation of a vanilla instrument involves multiple discount curves and projection for the baseline risk-neutral valuation and in addition to that large scale Monte Carlo simulations are needed to calculate

- Credit Valuation Adjustment (CVA)
- Debt Valuation Adjustment (DVA)
- Funding Valuation Adjustment (FVA)
- Capital Valuation Adjustment (KVA)
- Margin Valuation Adjustment (MVA) and Collateral Valuation Adjustment (ColVA)

for each counterparty. It is not reasonable anymore to value single trades separately from the other instruments on the portfolio. As a consequence, trades would be valued at portfolio level and these values then could be reallocated to the individual trades. (Green, 2016)

The basic idea of the current view on derivative valuation is to distinguish price from value. Value shows how much an instrument is worth to a given institution in that given portfolio, whereas Price means the exit price with respect to fair-value accounting that can be found in the books and records. As a result of the difference between these two concepts, a portfolio of derivatives is not worth the same for all market participants. (Ruiz, 2015a)

Based on this, one must argue that assuming that law of one price principle holds, all instruments should be traded on the same price on all markets otherwise there exists a trading strategy that makes profit without taking any risk, i.e. there is arbitrage opportunity. It is however, not trivial whether the no-arbitrage argument can be applied to OTC derivatives. The main argument is that the unsecured OTC derivatives can differ due to different terms in the ISDA agreement and also because
of counterparty risk, which is not the same even for derivatives with the same terms. 
(Green, 2016)

As seen in Table 4 the components of the value of a derivative transaction can vary based on the way it is traded. Here three cases of OTC trading are mentioned. Unsecured transactions, CSA covered trading and CCP clearing. The base valuation for each case is risk-neutral and it is adjusted to incorporate other risk factors that have impact on the value of the derivative. (Green, 2016)

<table>
<thead>
<tr>
<th>Table 4: Components of derivatives valuation (Table edited by author based on (Green, 2016))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-Crisis</strong></td>
</tr>
<tr>
<td><strong>Unsecured</strong></td>
</tr>
<tr>
<td>Risk-neutral price (LI-BOR discounting)</td>
</tr>
<tr>
<td>Hedging costs</td>
</tr>
<tr>
<td>CVA</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Profit</td>
</tr>
</tbody>
</table>

The price of a derivative aims to reflect that the derivatives are traded in a fair way irrespectively of the market environment, whereas the value incorporates the specific details of the transactions. Value-to-me (VtM) is the difference between the present value of expected future cash-flows (selling price) and the present value of expected future cost of manufacturing and risk managing the trade. (Ruiz, 2015a)

As stated before, the price of a derivative is the price at which it is traded. For OTC transactions it can be a result of negotiation, whereas exchange traded instruments are highly liquid and their prices are driven by supply and demand. This also confirms that the same OTC derivative can have different value for different market participants.

The value of derivatives can be contingent on the aspect and purpose of the valuation itself. One can distinguish accounting valuation, regulatory valuation and
trading valuation. The accounting measure is driven by accounting standards and principles (e.g. IFRS) and also by company law. It provides an objective measure for derivatives valuation so that the books and records reflect the fair value of assets and liabilities. From trading valuation perspective the key is not the objectivity rather that the valuation captures the risk factors properly so that the trader is able to risk manage their positions in their trading books. From regulatory point of view what matters the most is the capital requirement of the trades are correctly determined. These valuation perspectives can differ, a good example for this is that DVA is not allowed by regulators, but it has an impact on the book valuation of individual trades. (Green, 2016)

The recent findings on derivatives valuation aim to bring closer real markets to theory and to incorporate every relevant risk factor into valuation.
PART III:

INTRODUCTION TO xVA TERMS

This part first mentions several important considerations about xVA calculation in general. Then it gives detailed introduction to the majority of the different risk types with respect to OTC derivatives that are covered by the xVA components.
1 General considerations

The value of a derivative trade depends on a number of factors including risk factors and factors that are specific for the given institution and the portfolio. In order to define the accurate value of the transaction one must take every factor into consideration.

The most obvious from these factors is market risk, which is accounted for by simply calculating the risk neutral price. Another well-known factor is credit risk, which originates from the possibility that the counterparty might not be able (or willing) to fulfill the obligations of the contract, and this is considered by calculating the CVA. There is also funding risk, which is taken into consideration with FVA. Besides counterparty credit risk there is also a chance that the institution itself defaults or is not able to meet their obligations (DVA). One must take into account the impact of collateralization and margins via ColVA and MVA. On the top of these one must take regulatory capital costs into consideration, as well (KVA). There are further xVA terms that are not discussed in this thesis, such as TVA, which stands for tax value adjustment or RVA as replacement of derivative on downgrade.

Since this valuation framework is pretty recent, the components are often referred named differently by the market practitioners. For instance, Ruiz (2015a) considers FVA as the sum of ColVA, hedging valuation adjustment and a liquidity adjustment, whereas Gregory (2015) divides FVA into funding benefit adjustment (FBA) and funding cost adjustment (FCA) while considering ColVA and MVA separately.

Naturally these components are not independent from each other, as the underlying risk factors have cross effects. Besides the cross effects - as mentioned before - these valuation adjustments sometimes only convert one type of risk into another. A general structure of components is summarized in Figure 5.

There are additional factors that can be accounted for e.g. model uncertainty risk. Model risk management is significant given most of the components of the derivative transaction are based on a valuation model. Also regulators are focusing on model appropriateness. Therefore many institutions are putting more and more emphasis on model risk management, model validation and further controls regarding valuation and risk models. (Ruiz, 2015b)

Trade level and portfolio level valuation can differ as some components can offset each other. As pointed out before, a holistic approach with appropriate reallocation metrics can be more accurate, than trade level valuation when netting is allowed for the parties.

Calculating the accurate xVA terms for a given set of transactions is not simple,
given there are a great number of factors one must consider. There are internal and external factors. Internal factors can be determined by the given bank, whereas external factors are functioning as constraints for the modeling. Table 5 represents the potential factors divided into this two categories.

Table 5: External and internal factors regarding xVA (Based on author’s own categorization.)

<table>
<thead>
<tr>
<th>EXTERNAL</th>
<th>INTERNAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regulatory regime</td>
<td>Organizational structure and operation</td>
</tr>
<tr>
<td>Accounting rules</td>
<td>CSA agreements with counter-parties</td>
</tr>
<tr>
<td>Market performance (e.g. interest rates)</td>
<td>xVA modeling</td>
</tr>
<tr>
<td>Counterparty’s behavior</td>
<td>Portfolio optimization</td>
</tr>
<tr>
<td></td>
<td>Risk management</td>
</tr>
</tbody>
</table>

As xVA term are getting more and more significantly recognized, the need for proper xVA management is also rising. Therefore xVA desk started to appear where only CVA desks operated before. These xVA desks are responsible for managing xVA and in terms of functionality, these are part trading desk and part portfolio management groups. xVA management involves pricing, hedging, and allocation of xVA terms and xVA hedging costs to desks. Of course hedging requires computing first-order risk sensitivities like delta and vega. (Kenyon and Green, 2014)

According to Gregory (2015) xVA can be considered as an exotic option of all the trades in the trading book of the bank. Therefore in theory xVA can be hedged, but it is important to note that it cannot be hedged perfectly. In terms of system requirements, the biggest challenge is that Greeks need to be calculated, which can be extremely large-scale, and therefore requires optimization.

Kenyon and Green (2014) propose an analytic approach for xVA management, consisting of three main elements, namely (1) trade-level regression for pricing; (2) analytic computation of sensitivities using the chain rule for hedging and (3) global conditioning for xVA allocation. Gregory (2015) proposes using approximations, American Monte Carlo methods and grids for making the xVA calculations more efficient.
2 Counterparty credit risk

Counterparty (credit) risk is a general term for the risk that the counterparty, with whom one has entered into a financial contract, will fail to fulfill their side of the contractual agreement. This can be when the counterparty defaults, but not limited to this case. OTC derivatives historically carry large counterparty risk. Since the crisis, several actions have been taken to reduce this type of risk. It is not possible to eliminate counterparty risk perfectly, and risk mitigants do not remove counterparty risk rather convert it into other types of risk. Also these mitigants are partly responsible for the creation of additional xVA terms. This is summarized in Table 6. This also demonstrates the importance of integrated, central xVA management.

Credit Value Adjustment (CVA) quantifies the market value of counterparty credit risk. Based on Ruiz (2015b), to measure counter-party risk there are three main elements to consider, namely (1) exposure profile, (2) default probability (PD) and (3) loss given default (LGD).

2.1 Credit risk exposure profile

There are several exposure metrics that measure how much the bank may be owed in the event of a counterparty defaulting. Obviously CVA depends heavily on the exposure profile of the bank towards a given counterparty. Therefore first one should
Table 6: Effect of counterparty risk mitigants (Table edited by author based on (Gregory, 2015, p.28-29))

<table>
<thead>
<tr>
<th>MITIGANT</th>
<th>NEW RISK</th>
<th>XVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Netting</td>
<td>Legal risk</td>
<td>FVA,</td>
</tr>
<tr>
<td>Collateral</td>
<td>Operational risk, market risk and liquidity risk</td>
<td>ColVA</td>
</tr>
<tr>
<td>Other contractual clauses</td>
<td>Operational and liquidity risks</td>
<td>MVA</td>
</tr>
<tr>
<td>Hedging</td>
<td>Operational risk, additional market risk, wrong-way risk, systemic risk</td>
<td>MVA</td>
</tr>
<tr>
<td>Central counterparties</td>
<td>Operational risk, liquidity risks and systemic risk</td>
<td>KVA</td>
</tr>
</tbody>
</table>

define the metrics for quantifying the exposure profile.

When the counterparty defaults (or not able to fulfill their contractual obligations) there can be two different scenarios causing an asymmetry in CVA valuation. When the contract has positive value from the counterparty’s perspective (i.e they receive money) it does not make a difference whether or not the counterparty defaults, given the bank is still obliged to fulfill their end of the contract. On the other hand, when the counterparty owes the bank, the bank will have a claim on the positive value at the time of the default. This is described in Figure 6. It is possible that a fraction of the claim recovers, this is called Recovery Rate (RR) and is not included in the definition of exposure. (Gregory, 2012)

For unilateral case the exposure can be defined simply as $E_{CCR(t)} = max(value_t, 0)$. Given counterparty risk is by nature bilateral, this formula changes to a more complicated version. (Gregory, 2015)

![Figure 6: Impact of a positive or negative value of the contract (Figure edited by author based on (Gregory, 2015, p.110))](image-url)
It is one of the most difficult challenges to calculate the accurate exposure profile. There are a number of risk metrics that are often used in the industry. This thesis uses the metrics from Gregory (2015) as shown in Table 7. Gregory (2015) defines Expected Future Value (EFV), Potential Future Exposure (PFE), Expected Exposure (EE), Maximum PFE, Expected Positive Exposure (EPE), Negative Expected Exposure (NEE) and Effective Expected Positive Exposure (EEPE) based the original regulator definitions from Basel Committee on Banking Supervision (BCBS).

EEPE is a more conservative metric, used in regulatory capital calculations, because EPE may neglect very large exposures that are present for only a short time and it may underestimate short-dated transactions. These metrics are defined for netting sets i.e. the total number of transactions in scope after netting and including collateral amounts. (Gregory, 2015)

In the given formulas $P$ represents the price of the portfolio (or netting set) and $\Psi_t(P)$ is the estimate of the distribution of $P$ at time $t$. It is also important to note that these metrics are stochastic variables over the lifetime of the trade as depicted in Figure 7.

![Figure 7: Future exposure profile (Figure adapted and edited by author based on (Gregory, 2015, p.113))]
<table>
<thead>
<tr>
<th>Name</th>
<th>Explanation</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Future Value</td>
<td>The expected value of the netting set at some future point.</td>
<td>$EFV_t = \int_{-\infty}^{\infty} P\Psi_t(P) dP$</td>
</tr>
<tr>
<td>Expected Exposure</td>
<td>The average of all exposures of the netting set at some future point, where negative exposure is replaced with 0 value.</td>
<td>$EE_t = \int_{-\infty}^{\infty} \max(P, 0)\Psi_t(P) dP$</td>
</tr>
<tr>
<td>Negative Expected Exposure</td>
<td>The average of exposures of the netting set at some future point, where positive exposure is replaced with 0 value.</td>
<td>$NEE_t = \int_{-\infty}^{\infty} \min(P, 0)\Psi_t(P) dP$</td>
</tr>
<tr>
<td>Potential Future Exposure</td>
<td>Given a confidence level X, the value of the portfolio will be lower than the PFE in X percent of the cases at a certain time in the future.</td>
<td>$X = \int_{-\infty}^{PFE_t^X} \Psi_t(P) dP$</td>
</tr>
<tr>
<td>Maximum PFE</td>
<td>The worst-case exposure over the entire interval.</td>
<td>$PFE_{\text{max}} = \max PFE_t$</td>
</tr>
<tr>
<td>Expected Positive Exposure</td>
<td>How much the bank can be owed on average over the interval.</td>
<td>$EPE = \int_{-\infty}^{\infty} EE_t dt$</td>
</tr>
<tr>
<td>Expected Negative Exposure</td>
<td>How much the bank can owe on average over the interval.</td>
<td>$ENE = \int_{-\infty}^{\infty} NEE_t dt$</td>
</tr>
<tr>
<td>Effective Expected Exposure</td>
<td>A non-decreasing version of the EE profile.</td>
<td>$EEE_t = \max(EE_t, EE_t - 1)$</td>
</tr>
<tr>
<td>Effective expected positive exposure</td>
<td>The average of the EEE for a short time-frame, usually 1 year.</td>
<td>$EEPE = \frac{1}{n} (\sum_{i=1}^{n} EEE_{t_i}), t_i \leq 1$</td>
</tr>
</tbody>
</table>
To reduce counterparty risk, financial institutions add to their ISDA Master Agreements a Credit Support Annex (CSA) by which counterparties have to post collateral when the netting set is in the money. The CSA structure defines whether it is unilateral or bilateral, the type of the collateral and eligible assets, collateral ownership, interest on collateral and the possibility of rehypothecation (i.e. re-use the collateral by sale, repo, lending, or re-delivery).

According to Ruiz (2015b) there are five key CSA parameters that need to be defined when modeling collateralized exposures: (1) margining frequency, (2) threshold, (3) independent amount (IA) or initial margin (IM), (4) minimum transfer amount (MTA) and (5) rounding.

If collateral is present the basic calculation of current exposure profile slightly changes to the following:

$$E_{CCR}(t) = \max(value_t - C_{t-MPR}, 0)$$

where value is the value of the portfolio of trades in scope and $C$ is the value of the collateral held. MPR refers to the margin period of risk, which is the effective time between a counterparty stopping to post collateral and when all the underlying transactions have been closed-out and replaced. (Gregory, 2015)

Based on Ruiz (2015b) there are three main approaches to calculate exposure profile: (1) Brownian Monte Carlo, (2) historical Monte Carlo or (3) using simplifications and approximations. Brownian Monte Carlo uses risk-neutral measure and is based on market implied inputs, which can be difficult to obtain in some cases (e.g. illiquid assets). On the other hand historical Monte Carlo uses real-world measure based on historical data for the calculation, which assumes that the past events are somehow good indicators of the future.

### 2.2 Default probability and loss given default

Default Probability is an estimation of the probability that a counterparty will default at a given point in time. Real world and risk-neutral default probabilities differ and this has been a key point of interest of the increased importance of CVA in recent years. Real-world default probability is usually calculated based on historical default data via some the relevant credit ratings. A risk-neutral default probability is calibrated to available market instruments e.g. bonds or credit default swaps (CDSs). The risk neutral approach is widely used amongst market practitioners as accounting requirements and Basel III capital rules promote this method over the historical one.

There are several ways to define the credit spread based on different instruments, like single-name CDSs, asset swaps spreads, bond or loan prices and using some
proxy or mapping method can be also feasible.

Using the CDS market is the easiest and most intuitive given it directly defines credit spreads. There are several ways to calculate PD from CDS spreads. One simple approach is described in Gregory (2015), which assumes that the credit spread term structure is flat (credit spreads for all maturities are equal) and that CDS premiums are paid continuously. In this case the unconditional probability of default between any two sequential dates can be defined with the following formula:

\[ PD(t_{i-1}, t_i) = \exp\left( -\frac{s_{t_{i-1}} t_{i-1}}{LGD} \right) - \exp\left( -\frac{s_{t_i} t_i}{LGD} \right) \]

where \( PD(t_{i-1}, t_i) \) is the default probability between \( t_{i-1} \) and \( t_i \), \( s_t \) is the credit spread at time \( t \) and \( LGD \) is the assumed loss given default. This formula relies on the approximate relationship between the hazard rate and credit spread given by \( h = \frac{s}{LGD} \). To allow for a term structure of credit (for example, CDS premiums at different maturities) and a term structure of interest rates, one must choose some functional form for \( h \) hazard rate. This will be described in more details in the implementation part. (Gregory, 2015)

A more sophisticated process of calculating the PD from CDSs is referred to as bootstrapping. In this approach, PD is calibrated to not only one single liquid point, rather to the whole term structure of credit. If liquid enough, one could have CDSs for 3-month, 6-month, 9-month, 1-year, 2-year, etc., tenors. PD can be expressed in terms of the default intensity \( \lambda \). This is defined so that the survival probability (i.e. one minus the default probability) at time \( t \) can be expressed as \( S_t = \exp\left(-\int_0^t \lambda_u du\right) \). (Ruiz, 2015b)

This thesis also uses the term structure of CDS spreads to calibrate intensity based credit model and imply probability of default, which will be discussed in more details in the next part.

Loss Given Default provides an estimate of the percentage of loss over the total exposure, in the event of default. Equivalently, this is calculated as one minus the recovery rate. It is very common to assume flat recovery rate and therefore flat LGD, as well.

### 2.3 Bilateral Credit Value Adjustment

In the above the independence of the credit exposure and default probability was assumed, which results in neglecting wrong-way risk. Also, CVA and DVA are discussed separated from other xVA terms. This is a very important consideration since in practice xVA terms should not be dealt with separately, and possible overlaps should be considered proving the necessity of central xVA management again.
According to Gregory (2015) CVA can be considered as an adjustment on the risk-free value defined simply as $PV_{risky} = PV_{risk-free} + CVA^1$. This has a very important implication, namely, that for each trade the risk-free valuation and the CVA calculation can be separated from each other. This enables central CVA management and reallocation of counter-party risk chargers to the given trade. This is the foundation of the xVA desks.

CVA can be either unilateral or bilateral depending on whether the bank includes their own credit risk. The bilateral CVA can be considered as the difference between the cost of hedging the credit risk ($CVA_{asset}$) and the cost of funding the expected liabilities ($CVA_{liab}$). The difference between both is the bilateral CVA price:

$$bCVA = -(CVA_{asset} - CVA_{liab})$$

The liability side is usually referred to as Debt Value Adjustment (DVA). Market practitioners have different views about DVA. Some say that it should not be considered in pricing, given it provides a benefit in the balance sheet when a bank faces financial issues and because it cannot be monetized. (Ruiz, 2015b)

The standard formula for unilateral CVA in discrete form, based on Gregory (2015) can be given as:

$$uCVA = -LGD \sum_{i=1}^{m} EE(t_i)PD(t_{i-1}, t_i)$$

where $LGD$ is the ratio of the current exposure that we are expected to lose upon counter-party’s default, also referred to as $1 - RecoveryRate$, $EE$ is the discounted expected exposure for a future point in time and $PD$ is the marginal default probability between two consecutive time points. The bilateral CVA can be defined as:

$$bCVA = -(LGD_{CP} \sum_{i=1}^{m} EE(t_i)PD_{CP}(t_{i-1}, t_i) - LGD_{Bank} \sum_{i=1}^{m} NEE(t_i)PD_{Bank}(t_{i-1}, t_i))$$

where the DVA term is based on the negative expected exposure ($NEE$), the bank’s own default probability and its LGD. It is important to note that this formula implies that the correlation between exposure and default probability is negligible.

The dependency between exposure and default probability is called wrong-way risk (WWR), when the exposure increases with the probability of default, or right-way risk (RWR) when the exposure decreases with PD or collectively directional-way.

---

1In this thesis the CVA formula includes the appropriate + or - sign.
risk (DWR). This effect is common for the major asset classes, such as equity, foreign exchange (FX), commodities, and credit. A good example from Ruiz (2015b) is when a bank buys a put option from a counterparty with its own stock as the underlying. If the counterparty defaults, the stock will be worth zero and therefore the value of our long put position will be maximum, but the counter-party is not likely to be able to pay. Correct modeling of DWR is crucial for financial institutions. (Ruiz, 2015b)

There are several approaches to model DWR from very simple approximations to complex mathematical models, with both empirical and theoretical focus. Ruiz (2015b) mentions several of these approaches. The simplest is the Basel framework, which only takes WWR into account. This method increases the exposure metric by a constant $\alpha$ factor, for all counterparties and netting sets. Ruiz (2015b) proposes an empirical analysis, which is simple enough, calibrated to empirical data independently from trading book, leverages existing MC simulation engine without using abstract not observable variables.

There is of course a hazard rate approach, where the stochastic process of the hazard rate is correlated with the other underlying processes required for modeling the exposure. Default will be generated via the credit spread process and the resulting conditional $EE$ calculation will remain the same, but will only be calculated for paths on which there has been a default. The main difficulty of this approach is the calibration of correlation. Usually it can only be calibrated to historical data given the lack of direct market of correlation. Allowing WWR has an important implication on CVA calculation. The simple formula cannot be used anymore, instead the exposure becomes conditional upon default of the counterparty:

$$uCVA = -LGD \sum_{i=1}^{m} EE(t_i|t_i = \tau_{CP})PD(t_{i-1}, t_i)$$

where $\tau_{CP}$ is the default of the counterparty. (Gregory, 2015)

CVA calculation depends heavily on the level of collateralization between the counter-parts. For unsecured transactions both parties are fully exposed to counterparty risk. In theory, the amount of CVA and DVA are the same, only the direction is different:

$$CVA_A = DVA_B$$
$$CVA_B = DVA_A$$

Naturally these equations do not hold in practice, as banks operate different valuation models, which can result in mismatch of CVA and DVA for both parties causing collateral disputes. (Gregory, 2015)
For transactions with perfect collateralization, there should not be any CVA or DVA held. However, there is no such thing as a perfect CSA; there is always residual counter-party risk. What is more, CSAs usually define non-zero thresholds for minimum transfer amount (MTA). Also there is a so-called grace-period, which allows the party that has not fulfilled its collateral payment, to make the payment. Therefore when modeling counterparty-risk, usually a margin period of risk is included, which is most commonly set at ten days. CCP cleared transactions are similar to the bilateral contracts with CSA. The main difference is that CCPs variation margin agreements define a daily margin call. This reduces the residual risk. (Green, 2016)

So far in this section contract level CVA was mentioned, but in reality trades are handled in portfolios. In this case netting has a powerful effect and reduces CVA for a netting set\(^2\), which collects the transactions with a specific counterparty under the same netting agreement:

\[
CVA_{NS} \geq \sum_{i=1}^{n} CVA_i
\]

where \(CVA_{NS}\) is sum of stand-alone CVAs across all transactions in the netting set, and \(CVA_i\) is the CVA for the \(i^{th}\) transaction. According to Gregory (2015) to realize the benefits of portfolio level CVA a concept of incremental CVA was introduced, where the CVA of the \(i^{th}\) transaction is calculated based on its marginal effect on the netting set:

\[
CVA_{i}^{\text{incremental}} = CVA_{NS+i} - CVA_{NS}
\]

3 Funding risk

Funding risk is captured with FVA, which is defined as the costs and benefits from managing the collateral on hedges, which eliminate market risk from the unsecured transactions. FVA and CVA represent the cost of differing from a perfect CSA agreement. (Lichters et al., 2015)

3.1 Quantifying Funding Value Adjustment

Similarly to credit exposure, one can quantify the exposure profile for funding, as well. Also it is highly recommended to handle CVA, DVA and FVA with shared methodologies. As a thumb rule, collateral should be subtracted (added) to the exposure when received from (posted to) the counter-party. Based on Gregory (2015)

\(^2\)The CVA reduction is the CVA becoming less negative.
the differences between counterparty credit risk and funding exposure can be illustrated with the following formulas:

- \( E_{CR} = \max(value - VM - IM^R, 0) \) for credit exposure and
- \( EF = MTM - VM + IM^P \) for funding exposure.

Here \( value \) is the effective value of the contracts in scope at the default time of the counterparty, including the impact of risk mitigants (such as netting and collateral), \( VM \) is the variation margin and \( IM^R \) is the initial margin received. \( MTM \) is the funding cost or benefit and \( IM^R \) is the initial margin paid. (Gregory, 2015)

With respect to FVA, Gregory (2015) defines positive MTM as a derivative asset that cannot be monetized and hence it has to be funded resulting in a funding cost. On the other hand negative MTM creates a derivative liability that represents a loss that does not need to be paid immediately, resulting in a funding benefit.

FVA can be broken down into the sum of funding cost adjustment (FCA) and funding benefit adjustment (FBA). FVA can be quantified in a simple intuitive form based on exposure metrics from the previous section:

\[
FVA = - \left( \sum_{i=1}^{m} EE(t_i)FS_B(t_i)(t_i - t_{i-1}) + \sum_{i=1}^{m} NEE(t_i)FS_L(t_i)(t_i - t_{i-1}) \right)
\]

\[
FVA = -(FCA + FBA)
\]

where \( FS(t_i) \) is the forward funding spread for the time \( t_i \). Funding spread can be broken down to credit funding cost and funding liquidity risk premium. (Gregory, 2015)

An intuitive approach can be to define the funding spread as the sum of CDS spread and bond-CDS basis spread, which is illustrated in Figure 8.

If lending and borrowing rates are equal for uncollateralized contracts, the application of the FVA formula is equivalent to discounting at a rate including the funding spread. This method is a very simple way to include FVA, but it has some drawbacks. It assumes a symmetry between funding costs and funding benefits and does not allow any thresholds, minimum transfer amounts or one-way CSAs. To properly incorporate these into FVA, one has to model the EE/NEE consistently with the collateral term using the definition of funding exposure instead of credit exposure. (Gregory, 2015)
3.2 FVA and other xVA terms

As mentioned earlier, FVA can be broken down into the sum of funding cost adjustment (FCA) and funding benefit adjustment (FBA). This is similar to bCVA, which consists of CVA and DVA. Figure 9 illustrates this relationship between CVA and FVA in terms of exposure. Therefore overlap between the them has to be assessed accurately.

There is a tight link between FVA and DVA, as DVA can be thought of as the price of funding. In an ideal world with perfect information and infinite liquidity the cost of funding would equal to the cost of counterparty credit risk, i.e. the funding spread would be the same as the CDS spread. The spread over the risk-free rate at which an entity can borrow (unsecured) cash, is referred to as funding spread (Ruiz, 2015b)

This can be one of the reasons why FVA had so controversial critics and caused huge debates in quantitative finance. However the majority of market practitioners think that in one way or another the cost of funding unsecured derivative transactions should be included in pricing.

Gregory (2015) suggests two approaches to avoid double counting of the funding benefit:
- **CVA and symmetric funding** (CVA + FCA + FBA) or
- **Bilateral CVA and asymmetric funding** (CVA + DVA + FCA).

The first case relates more to the Basel III regulatory framework, which does not recognize DVA, given it cannot actually be monetized or properly hedged. On the other hand, the second case is more appropriate from accounting point of view, as the accounting rules require the recognition of DVA. Gregory (2015) shows that the common practice among the financial institutions is the CVA and symmetric funding.

Initial margin is not included with respect to FVA discussion, as it will be discussed separately as margin value adjustment (MVA). Funding costs (and benefits) in derivatives portfolios usually arise due to undercollateralisation or if the collateral cannot be rehypothecated (reused) and/or must be segregated, which will make it useless from a funding perspective.

![Figure 9: The relationship between bCVA and FVA (Figure edited by author based on (Gregory, 2015, p.339))](image-url)
4 Impacts of collateralization and initial margins

There are two value adjustments from collateralization and initial margin’s perspective. One is Margin Value Adjustment (MVA), which accounts for the cost of posting initial margin and any other overcollateralization such as the requirement for liquidity buffers. The other is ColIVA, which is an adjustment on collateralized transactions, accounting for deviating from a perfect collateral agreement.

4.1 Role of ColIVA

ColIVA covers the differences between collateral paid and/or received and the discount rate, as well as the option to chose the asset that will be posted as collateral. Gregory (2015) defines the appropriate rate at which perfectly collateralised transaction should be discounted as the rate at which collateral earns interest. Therefore ColIVA can be defined as:

$$ColVA = - \sum_{i=1}^{m} ECB(t_i)CS(t_i - t_{i-1})S(t_i)$$

where $ECB()$ is the expected collateral balance, $CS_t$ is the collateral spread and $S()$ is the joint survival probability. For uncollateralized transactions ColIVA will be zero, given ECB is zero in that case. In case of two-way collateralization with zero thresholds and small minimum transfer amounts, the ECB will be very close to the future MTM (i.e. EFV). The effect will be a change in the discounting rate by the collateral spread $CS_t$. When collateral will be posted in one direction only providing the threshold is zero then the ECB will be very close to the EE (or NEE). (Gregory, 2015)

The question of collateral optionality was raised as collateral arrangements tend to be rather flexible. A typical agreement will allow a range of cash and non-cash assets that are eligible for collateral posting including cash in different currencies, government and corporate bonds, equities, commodities and even mortgage-backed securities. For non-cash assets there will be a hair-cut defined. The optimal choice of collateral will be dependent on the return paid on the collateral, the hair-cut and the availability of the asset. Therefore collateral management has gain more and more importance in financial institutions.

4.2 Margin Value Adjustment

MVA covers the cost of IM and other financial resources required by a central counterparty (CCP) in case of central clearing. In case of bilateral contract it accounts
for the bilateral IM, which is mandatory from September 2016 for entities in the United States (US). It requires posting of IM by both parties. This IM must be segregated and cannot be rehypothecated. There are cases where collateral agreements require IM posting conditional upon some specific events e.g. credit rating downgrade. (Gregory, 2015)

As a formula, MVA can be written as:

$$MVA = -\sum_{i=1}^{m} EIM(t_i)(FC(t_i - s_{IM})(t_i - t_{i-1})S(t_i)$$

where $EIM()$ is the discounted expected initial margin and $FC()$ represents the funding cost of posting the initial margin with remuneration of $s_{IM}$. Similarly to CoIVA posting initial margin includes a cheapest-to-deliver optionality. (Gregory, 2015)

5 Regulatory capital risk

Capital requirements defined for market, credit, liquidity and operational risk. Therefore there is a chance of double-counting effects and not recognizing offsetting risks. Regulation can be inconsistent across regions. Although Basel III defines a global set of capital rules, the implementation is decided locally and may differ by region. Counterparty risk capital requirements has become more and more important in the recent years. Capital is used as a buffer against losses during turbulent periods and contributes to defining creditworthiness. Regulatory capital requirements determine the leverage ratio for the banks operations. (Gregory, 2015)

The Basel II framework provides two approaches for regulatory capital calculations. There is a simple method, which enables smaller banks to assess their risk exposure based on external ratings. The risk weighted asset (RWA) is defined as the exposure times the weight for the given category. The possible weights are 0%, 20%, 50%, 100% and 150% depending on the credit rating of the given counterparty. The more sophisticated one, the internal ratings-based (IRB) approach is based on on the bank’s own internal estimates of risk components. The factors defining the regulatory capital are the probability of default, loss given default, exposure at default and effective maturity. (Basel Committee on Banking Supervisions, 2006)

5.1 Calculating Exposure-at-Default

Exposure at default (EAD) can be calculated with one of the following methods under Basel II framework: (1) current exposure method (CEM), which is the simplest and most commonly used method, but will be replaced in 2017 by a more
risk-sensitive approach, namely the standardized approach for counterparty credit risk (SA-CCR) (Basel Committee on Banking Supervisions, 2014), (2) standardized method (SM), which is not commonly used and (3) internal model method (IMM), which is very advanced, but can be costly to implement. The first two are designed for those banks that do not have sophisticated internal models. CEM and SA-CCR approaches are based on predefined formulas, whereas the IMM is model-dependent relying on a complex set of modeling assumptions. (Basel Committee on Banking Supervisions, 2006)

EAD computation varies method by method. The CEM method defines EAD with the following formula:

$$EAD = CE + PFE_{remaining}$$

where $CE$ is the current exposure i.e. the positive MTM of the portfolio and $PFE_{remaining}$ is the estimated amount of the PFE over the remaining life of the contract. CEM is able to include netting and collateral effects in a very simple way. The CEM for collateralized netting set $n$ can be given as

$$CEM_{NS} = \max(\max(\sum_{i=1}^{n} MTM_i, 0) - C_A, 0) + (0.4 + 0.6 NGR) \sum_{i=1}^{n} PFE_{remaining}^i$$

where $NGR$ stands for net gross ratio, which represents the current impact of netting in percentage and $C_A$ is the market value of volatility adjusted collateral. The formula for calculating NGR can be defined as

$$NGR = \frac{\max(\sum_{i=1}^{n} MTM_i, 0)}{\sum_{i=1}^{n} \max(MTM_i, 0)}$$

(Gregory, 2015)

The IMM is a risk sensitive approach where both EAD and the maturity adjustment factor is generated by the bank’s internal models. Generally the EAD under IMM is given as:

$$EAD = \alpha \times EEPE$$

where $EEPE$ stands for effective expected positive exposure and the standard value for $\alpha$ is 1.4 or more. (Gregory, 2015)

SA-CCR approach has been developed to eliminate the shortcomings of CEM (and also SM) without adding extra complexity to the method. The new SA-CCR method allows better representation of maturity, a more accurate recognition of netting, better treatment of collateral in terms of risk sensitivities and possibility of negative MTM. EAD can be defined under SA-CCR as the following:

$$EAD = \alpha(RC + PFE)$$
where $RC$ is the replacement cost, $PFE$ is linked to an $AddOn = SF_iSD_i$ factor, $SF_i$ being a supervisory factor depending on the asset class and accounts for a loss over a one-year (uncollateralised) or shorter (collateralized) period, and $SD_i$ representing the duration. According to Gregory (2015), when collateral is present the replacement cost can be defined as:

$$RC = \max(V - C, TH + MTA - NICA, 0)$$

where $NICA$ is the net independent collateral amount, which will be available in default, $TH$ stands for account thresholds and $MTA$ is the minimum transfer amount.

### 5.2 Quantifying Capital Value Adjustment

The increased focus on regulatory capital resulted in higher capital costs, which can be considered another sort of funding and is analogous with FVA and MVA calculation. Therefore capital value adjustment (KVA) has also gained momentum in the recent years. According to Gregory (2015) there are three major forms of capital charges in connection with OTC derivatives. Default risk capital charge is related to counterparty risk and is also known as CCR capital charge. CVA capital charge aims to account for the MTM volatility from changes in CVA driven by credit spread movements without any real default event. Market risk capital charge aims to capitalize the risks from market movements. However banks usually hedge their market risks, therefore are not expected to have significant charges in this domain.

It is necessary to know the capital profile of the given bank to calculate KVA. It can be generated in a similar way as for MVA. KVA can be given as:

$$KVA = -\sum_{i=1}^{m} EC(t_i)CC(t_i)(t_i - t_{i-1})S(t_i)$$

where $EC()$ represents the discounted expected capital profile. The discount rate however is not trivial. Historically it is discounted by the cost of capital, based on the bank’s own dividend policy or CAPM. On the other hand many argue that it would be more sensible to discount by the risk-free rate or not include discounting at all. KVA is computationally intensive given it should be calculated for a bank’s whole portfolio simultaneously. The default risk capital charge is additive across netting sets, whereas the CVA capital charge should be calculated at portfolio level. Also KVA relies on some assumptions about the future regulatory era, which in fact is impossible to foresee. Therefore it is highly subjective. (Gregory, 2015)
PART IV:

xVA IMPLEMENTATION

This part declares different assumptions of implementing xVA calculations with respect to the analysis presented in this thesis and it also gives a primer on the results of the calculations along with its limitations.
1 Implementation specifications

As already mentioned, xVA calculation is computationally intensive as one needs to simulate the exposure profile for all future dates. In practice xVA calculation relies heavily on numerical methods like Monte Carlo simulation. Therefore it is of high importance to build a proper Monte Carlo engine and evaluate its precision. To have a reasonable output it is also significant to collect the appropriate input data from the market and use it with correct modeling assumptions. (Green, 2016)

The general work-flow scheme includes collection of inputs, calculation and evaluation of results. This can be seen in more details on Figure 10.

1.1 Software choice

This thesis uses R programming language for calibration, calculation and data visualization purposes. The great advantage of R over other statistical and mathematical software is that it is completely free and open-source with large developer community and lot of resources. It is very easy to use and there are already implemented packages in this topic. For instance, this thesis used RQuantlib, stats, cubature packages for calibration of market models, sde and xVA for the xVA calculation. For data visualization plotly and ggplot2 packages were used.

Figure 10: xVA implementation work-flow (Figure edited by author based on (Green, 2016, p.394))
1.2 Modeling assumptions

To define the specifications of xVA calculation, first thing to do is to decide some basic, but important questions. The calculation depends on whose perspective it is done from and of course on the counterparty, as well. Also it is a significant factor how they handle netting and what sort of initial agreements they have (e.g. ISDA Master Agreement with CSA). Besides this, the transaction’s specifications also have a huge impact on xVA calculation:

- Bank: Morgan Stanley,
- Counterparty: Other investment bank,
- Trading mechanism: OTC,
- CSA agreement: Yes,
- Trade: Vanilla interest rate swap.

Another important question is the discounting. This thesis uses OIS rate as the proxy of risk-free rate. This approach has become very common since the crisis, as LIBOR includes a risk premium for unsecured funding over OIS.

This thesis assumes a netting set of two vanilla interest rate swaps with kinetics based on Example 1 in (Basel Committee on Banking Supervisions, 2014, p.22). The relevant contractual terms can be found in Table 8. It is assumed that these swaps are already live trades at the time of the calculations. It is important to note that the fixed rate of these swaps are not the fair swap rates therefore the value of the swap will not be zero at this time point. The swaps in scope of the analysis

Table 8: Specifications of the interest rate swaps

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>SWAP#1</th>
<th>SWAP#2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nature</td>
<td>Interest rate swap</td>
<td></td>
</tr>
<tr>
<td>Residual maturity</td>
<td>10 years</td>
<td>4 years</td>
</tr>
<tr>
<td>Base currency</td>
<td>USD</td>
<td>USD</td>
</tr>
<tr>
<td>Notional</td>
<td>$10,000</td>
<td></td>
</tr>
<tr>
<td>Pay Leg</td>
<td>Fixed</td>
<td>Floating</td>
</tr>
<tr>
<td>Fixed rate</td>
<td>0.5%</td>
<td>0.3%</td>
</tr>
</tbody>
</table>

are subject to a bilateral margin agreement with specifications based on Example 5 in (Basel Committee on Banking Supervisions, 2014, p.29) as seen in Table 9. The
processing organization (PO) in this thesis is Morgan Stanley with the assumption that minimum transfer amount is $50, initial margin posted is $100 and margin threshold is $1 for the PO.

Table 9: Specifications of the margin agreements with each counterparty

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>MORGAN STANLEY &amp; CITI</th>
<th>MORGAN STANLEY &amp; GOLDMAN SACHS</th>
<th>MORGAN STANLEY &amp; JPMORGAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Margin frequency</td>
<td>Monthly</td>
<td>Monthly</td>
<td>Monthly</td>
</tr>
<tr>
<td>MPR (days)</td>
<td>10</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Threshold</td>
<td>$2</td>
<td>$1</td>
<td>$1</td>
</tr>
<tr>
<td>Minimum Transfer Amount</td>
<td>$40</td>
<td>$20</td>
<td>$40</td>
</tr>
<tr>
<td>Initial Margin posted by CP</td>
<td>$150</td>
<td>$130</td>
<td>$120</td>
</tr>
</tbody>
</table>

This thesis implements CVA and DVA terms along with symmetric funding, i.e. both FCA and FBA are accounted for. Another assumption is that the collateral spread (i.e. the spread paid on the collateral with respect to the risk-free rate) is zero, therefore ColVA will be zero and not calculated in this thesis.

For FVA calculation the funding spreads are derived with the approach from Figure 8, i.e. the funding spread equals the bond yields\(^3\) of the PO minus the OIS spread as shown in Table 10. Here it is assumed that the funding spread of borrowing and lending rates are the same for each tenor. Additional assumptions for

Table 10: Morgan Stanley corporate bond yields and funding spreads as of 25-Oct-2016

<table>
<thead>
<tr>
<th>TENOR</th>
<th>BOND YIELD</th>
<th>FUNDING SPREAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1Y</td>
<td>1.68%</td>
<td>1.057%</td>
</tr>
<tr>
<td>2Y</td>
<td>2.94%</td>
<td>2.225%</td>
</tr>
<tr>
<td>3Y</td>
<td>2.16%</td>
<td>1.373%</td>
</tr>
<tr>
<td>5Y</td>
<td>3.64%</td>
<td>2.709%</td>
</tr>
<tr>
<td>10Y</td>
<td>3.92%</td>
<td>2.702%</td>
</tr>
</tbody>
</table>

\(^3\)Downloaded from Morningstar
the KVA calculation are summarized in Table 11. The IMM regulatory framework is assumed, given it the actual framework used by Morgan Stanley. (Morgan Stanley, 2016) Return on Capital (ROC)\(^4\) for the PO is assumed to be 3.37%. To calculate

Table 11: Relevant assumptions about counterparts for KVA calculation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Citi</th>
<th>Goldman Sachs</th>
<th>JP Morgan</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD</td>
<td>9%</td>
<td>8%</td>
<td>7%</td>
</tr>
<tr>
<td>LGD</td>
<td>60%</td>
<td>60%</td>
<td>60%</td>
</tr>
<tr>
<td>Fitch Ratings</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>MVA days</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>MVA percentile</td>
<td>99%</td>
<td>99%</td>
<td>99%</td>
</tr>
</tbody>
</table>

the xVA amount for a given product with a given counterparty one needs to model the interest rate term structure, and the specific risk factors that are associated with the given product. This thesis focuses on OTC interest rate swaps, therefore a proper interest rate model should be chosen and the credit risk of the counterparty has to be captured correctly, as well.

The key risk factor in the xVA calculation is the interest rate risk, so the calculation depends heavily on the choice of interest rate model. All other models should be chosen in a way to fit with the chosen interest rate model. Interest rates are usually considered to be stochastic, so the choice of measure influences the form of all other stochastic processes. (Green, 2016)

Green (2016) collected several potential choices for interest rate modeling in the xVA framework as shown on Figure 11. This thesis uses one-factor Hull-White model calibrated to the swaption volatility where the term structure of interest rates is given by OIS rates. Given that this thesis focuses on interest rate swaps we do not require our model to handle volatility surface accurately.

Besides interest rate modeling, finding the proper credit model is also essential for the xVA calculation. This thesis uses the intensity-based affine one-factor Cox-Ingersoll-Ross (CIR) model to simulate the credit default probabilities. The CIR model is calibrated to Credit Default Swap (CDS) spreads of the given counterparty. The modeling choices are summarized in Table 12.

\(^4\)Downloaded from GuruFocus.com
Table 12: Summary of model choices

<table>
<thead>
<tr>
<th>Risk Factor</th>
<th>Model Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate:</td>
<td>One-factor Hull-White for short rate</td>
</tr>
<tr>
<td>Credit default probability:</td>
<td>One-factor Cox-Ingersoll-Ross for default intensity</td>
</tr>
<tr>
<td>IR-Credit correlation:</td>
<td>Assumption of independence</td>
</tr>
</tbody>
</table>

Figure 11: Interest rate models and modeling frameworks (Figure edited by author based on (Green, 2016, p.287).)

2 xVA calculation

The goal of the calculation is to see how much the value of the same netting set changes across counterparts of the contracts, i.e. how much will the same netting set be worth for Morgan Stanley if the trades are done with Citi Group or Goldman Sachs or JP Morgan? Based on Ruiz (2015b) xVA calculation can be broken down into three steps, namely (1) calibration of models, (2) simulation and valuation and (3) calculating xVA integrals. This approach is used in this thesis, as well.
2.1 Step 1: Model calibration

Brigo and Alfonsi (2004) conclude that the calibration of the interest-rate and the credit model can, in fact, be done separately, without loss of efficiency. Therefore this thesis also separates the calibration of the one-factor Hull-White model for interest rates and the Cox-Ingersoll-Ross model for credit.

2.1.1 Input sources for calibration

The necessary input data for the interest rate and credit model calibration is derived from the Bloomberg terminal as of 25-October-2016. The two most important inputs for the Hull-White interest rate model calibration are: (1) USD OIS curve, (2) USD swaption ATM Black implied volatility (IndexTenor:6M, discounting:OIS). For the CIR credit model, the USD denominated senior CDS spread curve is used as input for the entities in scope: (1) Morgan Stanley, (2) Goldman Sachs Group Inc., (3) JP Morgan Chase & Co. and (4) Citigroup Inc.

2.1.2 Interest rate model

As the first step, the parameters of the one-factor Hull-White model should be calibrated. Based on Hull (2012), the dynamics of the short rate under $Q$ risk neutral measure can be defined as the following stochastic differential equation (SDE):

$$dr = (\theta(t) - ar)dt + \sigma dW^Q$$

or

$$dr = a(\frac{\theta(t)}{a} - r)dt + \sigma dW^Q$$

where $W$ is a standard Brownian motion, $a$ is the mean reversion and $\sigma$ is the volatility parameter for the short rate $r$, which is the rate that applies to an infinitesimally short period of time at a given point of time $t$. It is also known as instantaneous short rate. Based on Hull (2012) the bond prices can be written in the one-factor Hull-White model as follows:

$$P(t, T) = A(t, T) \exp (-B(t, T)r(t))$$

where $P(t, T)$ is the price of a zero-coupon bond at time $t$ that pays $1$ at time $T$. $A(t, T)$ and $B(t, T)$ can be expressed in the following analytic forms:

$$B(t, T) = \frac{1 - \exp (-a(T - t))}{a}$$

$$\ln A(t, T) = \ln \frac{P(0, T)}{P(0, t)} + B(t, T)F(0, t) - \frac{1}{4a^3}\sigma^2(\exp (-aT) - \exp (-at))^2(\exp (2at) - 1)$$
where $F(0, t)$ is the instantaneous forward rate for a maturity $t$ as seen at time zero.

The analytic tractability of HullWhite model is exploited in this thesis, however it is important to note that the Hull-White model can also be represented in the form of a trinomial tree.

This thesis uses the calibration tool-kit implemented in the $RQuantlib$ library in R using the $BermudanSwaption()$ method. The inputs can be seen in Table 13 and Table 14 and in Figure 12.

The $R$ library works as an intermediary layer between the user and the original C++ library’s methods. For instance, $BermudanSwaption()$ method calls the $calibrateModel()$ method in $Quantlib$ library of C++, which is based on the Levenberg Marquardt minimization algorithm and it exports the output back to R from either $bermudanWithRebuiltCurveEngine()$ or $bermudanFromYieldEngine()$, depending on the inputs. The inputs used in this thesis yield the output from $bermudanFromYieldEngine()$, which constructs the zero curve with the chosen spline interpolation. This thesis used the analytic Hull-White calibration, which implements a swaption engine using Jamshidian’s decomposition algorithm for calibrating $a$ and $\sigma$ parameters for the Hull-White model. (Gurrieri et al., 2009)

The calibration resulted in $a = 0.0633$ and $\sigma = 0.0239$ parameters.

Table 13: USD OIS curve as of 25-Oct-2016

<table>
<thead>
<tr>
<th>Tenor</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D</td>
<td>0.41%</td>
</tr>
<tr>
<td>1M</td>
<td>0.416%</td>
</tr>
<tr>
<td>3M</td>
<td>0.495%</td>
</tr>
<tr>
<td>6M</td>
<td>0.558%</td>
</tr>
<tr>
<td>9M</td>
<td>0.594%</td>
</tr>
<tr>
<td>1Y</td>
<td>0.623%</td>
</tr>
<tr>
<td>2Y</td>
<td>0.715%</td>
</tr>
<tr>
<td>3Y</td>
<td>0.787%</td>
</tr>
<tr>
<td>5Y</td>
<td>0.931%</td>
</tr>
<tr>
<td>10Y</td>
<td>1.218%</td>
</tr>
</tbody>
</table>

---

5For more details see (Gavin, 2016)
6For more details see (Brigo and Alfonsi, 2004)
Table 14: USD ATM swaption Black volatility as of 25-Oct-2016

<table>
<thead>
<tr>
<th>In/For</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34.69%</td>
<td>38.91%</td>
<td>42.35%</td>
<td>41.03%</td>
<td>39.15%</td>
</tr>
<tr>
<td>2</td>
<td>41.49%</td>
<td>43.13%</td>
<td>42.29%</td>
<td>40.82%</td>
<td>38.99%</td>
</tr>
<tr>
<td>5</td>
<td>43.20%</td>
<td>42.10%</td>
<td>38.96%</td>
<td>37.54%</td>
<td>36.29%</td>
</tr>
<tr>
<td>7</td>
<td>39.16%</td>
<td>38.45%</td>
<td>36.20%</td>
<td>35.31%</td>
<td>34.34%</td>
</tr>
<tr>
<td>10</td>
<td>34.72%</td>
<td>33.88%</td>
<td>33.22%</td>
<td>32.83%</td>
<td>31.88%</td>
</tr>
</tbody>
</table>

Figure 12: Black implied volatility of USD ATM swaptions as of 25-Oct-2016
2.1.3 Credit model

After the interest rates model, the credit model has to be calibrated to CDS spreads, which are shown in Table 15. This thesis uses Cox-Ingersoll-Ross model, which follows mean reverting, square-root dynamics for modeling the $\lambda$ default intensity process. The SDE of the CIR model is as follows:

$$d\lambda_t = \kappa(c - \lambda_t)dt + \sigma\sqrt{\lambda_t}dW_t$$

and

$$\lambda_t = \mathbb{P}(t \leq \tau < t + dt | \tau > t)$$

where $W_t$ is a standard Brownian motion, $\sigma$ is the volatility parameter, $c$ is the level where the process converges back to, with the rate of $\kappa$ and $\tau$ is the default time. The CIR model has some useful features, most importantly it always generates positive default intensity if the Feller condition holds, i.e. $\sigma^2 \leq 2\kappa c$. Under the CIR model, conditional survival probability at time $s$-providing that the name has not defaulted before $t$- can be given with a closed formula:

$$\mathbb{P}(\tau > s | \mathcal{F}_t) = \mathbb{E}[\exp(-\int_t^s \lambda_u du)] = \exp(a(t,s) + b(t,s)\lambda_t)$$

where for $\gamma = 0.5\sqrt{\kappa^2 + 2\sigma^2}$

$$b(t,s) = \frac{-\sinh(\gamma(s-t))}{\gamma \cosh(\gamma(s-t)) + 0.5\kappa \sinh(\gamma(s-t))}$$

and

$$a(t,s) = \frac{2\kappa c}{\sigma^2} \log \frac{\gamma \exp(0.5\kappa(s-t))}{\gamma \cosh(\gamma(s-t)) + 0.5\kappa \sinh(\gamma(s-t))}$$

(Fang et al., 2012)

The calibration of CIR model parameters can be done via the Least Squares Estimation. In case of fair CDS spread $S$, the value of the default leg and the value

<table>
<thead>
<tr>
<th>TENOR</th>
<th>MORGAN STANLEY</th>
<th>CITI</th>
<th>GOLDMAN SACHS</th>
<th>JPMORGAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1Y</td>
<td>0.356%</td>
<td>0.260%</td>
<td>0.373%</td>
<td>0.266%</td>
</tr>
<tr>
<td>2Y</td>
<td>0.481%</td>
<td>0.402%</td>
<td>0.484%</td>
<td>0.348%</td>
</tr>
<tr>
<td>5Y</td>
<td>0.872%</td>
<td>0.771%</td>
<td>0.904%</td>
<td>0.613%</td>
</tr>
<tr>
<td>7Y</td>
<td>1.162%</td>
<td>1.054%</td>
<td>1.202%</td>
<td>0.841%</td>
</tr>
<tr>
<td>10Y</td>
<td>1.387%</td>
<td>1.224%</td>
<td>1.425%</td>
<td>1.045%</td>
</tr>
</tbody>
</table>
of the premium leg are the same at time $t$. Therefore $S_t(T)$ can be written as the fraction of the default leg and the premium leg:

$$S_t(T) = \frac{\ell R_t(T)}{V_t(T)}$$

where $R_t(T)$ is the pre-default value of a unit recovery payment at $t$, $V_t(T)$ is the value of premium payments until default and $\ell$ is the loss given default. (Fang et al., 2012)

The objective function for the least squares method to determine the optimal set of model parameters can be given as:

$$\arg \min_{\theta} \sum_i [S(T_i)V_0(T_i; \theta) - \ell R_0(T_i; \theta)]$$

where $\theta = (\kappa, c, \sigma, \lambda_0)$, $S(T_i)$ is the CDS spread for the given name and for every $T_i = 1Y, 2Y...$,

$$V_0(T_i) = \int_0^T \exp(-rs) \exp(a(0,s) + b(0,s)\lambda_0)ds$$

and

$$R_0(T_i) = \exp(-rT)(1 - \exp(a(0,T) + b(0,T)\lambda_0))$$

$$+ r \int_0^T \exp(-rs)(1 - \exp(a(0,s) + b(0,s)\lambda_0))ds$$

(Fang et al., 2012)

The calibration was implemented in R in a semi-manual approach, given the currently available libraries were not appropriate for minimizing the objective function defined earlier. Therefore the parameters found by trial and error, so they do not give global minimum, only local. Table 16 contains the selected parameters and their calibration errors. The calibration had to be separate for every entity in scope to have different parameters in the CIR model, which enables to calculate different probability of default for each name.

### 2.1.4 Correlation of risk factors

The correlation of the underlying risk factors (a.k.a right way risk and wrong way risk) for xVA modeling has somewhat divided practitioners. On one hand it can have a sizable impact on the xVA numbers in certain cases and correctly modeling the dependency structure is key for exposure measurement. On the other hand -as Stein et al. (2010) argues- there are only limited options to calibrate the correlation to the market, given there are not many derivatives that directly access this correlation. Although is possible to analyze historical data, it may cause inconsistencies with the pricing measure.
Table 16: Calibrated parameters for the CIR model

<table>
<thead>
<tr>
<th>Param.</th>
<th>Morgan Stanley</th>
<th>Citi</th>
<th>Goldman Sachs</th>
<th>JPMorgan</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>0.300</td>
<td>1.400</td>
<td>1.200</td>
<td>0.300</td>
</tr>
<tr>
<td>$c$</td>
<td>0.030</td>
<td>0.020</td>
<td>0.180</td>
<td>0.020</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.100</td>
<td>0.150</td>
<td>0.220</td>
<td>0.005</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Calib.err.</td>
<td>8.43e-05</td>
<td>2.05e-04</td>
<td>3.09e-04</td>
<td>1.13e-06</td>
</tr>
</tbody>
</table>

Ruiz (2015b) highlights the difference between dependency and correlation i.e. correlation is only one measure of dependency. For example, two variables with a dependency structure modeled by a correlation parameter and a copula structure. If the copula is changed without modifying the correlation parameter, the dependency structure will be different but the linear correlation remain the same. Similarly to simplifying the distribution of a variable to a single number, simplifying the whole dependency framework to a linear correlation number would not be sensible. (Ruiz, 2015b)

As modeling dependency structure is a complex problem both from a methodology and a computational perspective, this thesis assumes the independence of the short rate and the default intensity processes.

2.2 Step 2: Simulation and valuation

This subsection presents the exposure profile and PD calculation for the given entities in scope.

2.2.1 Exposure profile

The exposure profile of the netting set consisting the aforementioned two swaps (with contract specification and CSA agreements described in Table 8 and Table 9) was calculated for three scenarios, when Morgan Stanley does the trade with (1) Citi Group, (2) Goldman Sachs, (3) JP Morgan Chase.

The calculations were done in R for 1000 path, using the `CalcSimulatedExposure()` method from `xVA` library as the baseline for the calculations. This method implements the discrete version of both collateralized and uncollateralized EE, NEE, PFE\(^7\) and EEE exposure metrics described earlier in Table 7 using the Hull White

---

\(^7\)Assuming 90% confidence level
no-arbitrage model for short rates. The expected exposure (EE) profile is shown in Figure 13. The uncollateralized EE is not zero in the beginning of the exposure profile as the exposure profiles are not calculated at inception, rather the trades are assumed to be live.

![Figure 13: Uncollateralized expected exposure profile of the netting set](image)

2.2.2 Probability of default and loss given default

The risk-neutral probability of default (PD) of the names in scope between any two sequential dates, is one of the key factors for xVA calculation. It is important to emphasize that this is an unconditional probability, i.e. the probability of a name defaulting between $t$ and $t + \delta t$ is not conditional on the name surviving up to $t$.\cite{Gregory2015}

As mentioned before, the CIR model is used for credit modeling, which has the great advantage of having an analytic solution for the conditional survival probability until time $s$, given that the name has not defaulted before $t$, given $t \leq s$. Based on Fang et al. (2012) conditional survival probability can be given as:

$$P_{\text{cond.surv}} = \mathbb{P}(\tau > s | \mathcal{F}_t) = \exp(a(t, s) + b(t, s)\lambda_t)$$

The unconditional default probabilities between two consecutive dates can be derived from the conditional survival probability based on Bayes’s formula.\footnote{For more details see (Papoulis, 1991, p.65-79).} The survival probability for $t \leq s$ is

$$P_{\text{un.cond.surv}} = \mathbb{P}(\tau > s) = \mathbb{P}(\tau > s | \mathcal{F}_t)\mathbb{P}(\mathcal{F}_t)$$
from which the cumulative probability of default can be given as

$$cumPD(t_i) = 1 - P_{un}\text{cond.surv}(t_i)$$

and the marginal default probability between two consecutive dates can be written as

$$PD_m(t_i, t_{i+1}) = cumPD(t_i) - PD(t_{i-1}, t_i), i = 2..$$

This marginal default probability is unconditional. (APPENDIX 12A. in Gregory (2015))

The unconditional default probabilities for a 10-year interval with 0.5 year steps, starting from 0 with the calibrated CIR parameters for the entities in scope are shown in Table 17 and the cumulative default probabilities are illustrated with Figure 14. The cumulative default probabilities are smoother for those names that have smaller σ parameters. As expected, the four names are very similar in terms of default probabilities.

Ruiz (2015b) points out that the loss given default (LGD) can only be calibrated historically, given one would need a second asset besides CDS to calibrate the LGD to market. This thesis assumes flat 60%. It is a common practice to assume fixed 40% recovery rate (RR), which result in \(LGD = 1 - RR = 60\%\).

Figure 14: One realization of cumulative probability of default for the four entities over 10 years
Table 17: One realization of probability of default for the four entities over 10 years

<table>
<thead>
<tr>
<th>$t_i$</th>
<th>Morgan Stanley</th>
<th>Citi</th>
<th>Goldman Sachs</th>
<th>JP Morgan</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.153%</td>
<td>0.316%</td>
<td>0.260%</td>
<td>0.118%</td>
</tr>
<tr>
<td>0.5</td>
<td>0.339%</td>
<td>0.658%</td>
<td>0.544%</td>
<td>0.240%</td>
</tr>
<tr>
<td>1</td>
<td>0.645%</td>
<td>1.145%</td>
<td>0.955%</td>
<td>0.462%</td>
</tr>
<tr>
<td>1.5</td>
<td>0.997%</td>
<td>1.553%</td>
<td>1.333%</td>
<td>0.674%</td>
</tr>
<tr>
<td>2</td>
<td>1.399%</td>
<td>2.081%</td>
<td>1.783%</td>
<td>0.971%</td>
</tr>
<tr>
<td>2.5</td>
<td>1.839%</td>
<td>2.497%</td>
<td>2.159%</td>
<td>1.246%</td>
</tr>
<tr>
<td>3</td>
<td>2.308%</td>
<td>3.037%</td>
<td>2.652%</td>
<td>1.596%</td>
</tr>
<tr>
<td>3.5</td>
<td>2.789%</td>
<td>3.430%</td>
<td>3.028%</td>
<td>1.915%</td>
</tr>
<tr>
<td>4</td>
<td>3.349%</td>
<td>3.987%</td>
<td>3.486%</td>
<td>2.304%</td>
</tr>
<tr>
<td>4.5</td>
<td>3.908%</td>
<td>4.349%</td>
<td>3.866%</td>
<td>2.652%</td>
</tr>
<tr>
<td>5</td>
<td>4.442%</td>
<td>4.891%</td>
<td>4.293%</td>
<td>3.064%</td>
</tr>
<tr>
<td>5.5</td>
<td>5.067%</td>
<td>5.252%</td>
<td>4.696%</td>
<td>3.432%</td>
</tr>
<tr>
<td>6</td>
<td>5.593%</td>
<td>5.786%</td>
<td>5.108%</td>
<td>3.861%</td>
</tr>
<tr>
<td>6.5</td>
<td>6.224%</td>
<td>6.147%</td>
<td>5.514%</td>
<td>4.239%</td>
</tr>
<tr>
<td>7</td>
<td>6.797%</td>
<td>6.666%</td>
<td>5.902%</td>
<td>4.677%</td>
</tr>
<tr>
<td>7.5</td>
<td>7.392%</td>
<td>7.021%</td>
<td>6.302%</td>
<td>5.061%</td>
</tr>
<tr>
<td>8</td>
<td>7.920%</td>
<td>7.512%</td>
<td>6.659%</td>
<td>5.504%</td>
</tr>
<tr>
<td>8.5</td>
<td>8.545%</td>
<td>7.870%</td>
<td>7.069%</td>
<td>5.886%</td>
</tr>
<tr>
<td>9</td>
<td>9.132%</td>
<td>8.346%</td>
<td>7.401%</td>
<td>6.333%</td>
</tr>
<tr>
<td>9.5</td>
<td>9.714%</td>
<td>8.705%</td>
<td>7.854%</td>
<td>6.716%</td>
</tr>
</tbody>
</table>
2.3 Step 3: xVA integrals

This thesis used \textit{CalcVA()} and \textit{CalcKVA()} methods from \textit{xVA} library in R as the starting point for the calculations with the necessary inputs from previous sections. The xVA amounts are calculated for the netting set assuming three different counterparties for comparison. Also it is important to note that CVA and DVA are calculated with both FBA and FCA, however according to Gregory (2015) the most common approach is to only account for CVA with symmetric funding. As already noted, ColVA is not calculated, as this thesis assumes that the collateral spread is zero and therefore ColVA would be zero.

CVA is calculated based on the discounted collateralized EE, the LGD and default probability of the counterparty. DVA is the opposite of CVA in theory, it is based on the discounted NEE with the LGD and PD of the processing organization. However in practice the absolute value of these two amounts are not the same as seen in Table 18.

Table 18: CVA and DVA for the three counterparties (in thousand dollars)

<table>
<thead>
<tr>
<th>Term</th>
<th>Citi</th>
<th>Goldman Sachs</th>
<th>JP Morgan</th>
</tr>
</thead>
<tbody>
<tr>
<td>CVA</td>
<td>$-520.10</td>
<td>$-400.20</td>
<td>$-356.20</td>
</tr>
<tr>
<td>DVA</td>
<td>$537.30</td>
<td>$584.90</td>
<td>$580.90</td>
</tr>
<tr>
<td>bCVA</td>
<td>$17.20</td>
<td>$184.70</td>
<td>$224.70</td>
</tr>
</tbody>
</table>

The FCA and FBA are calculated similarly to CVA and DVA, the only difference is that instead of PD from credit spreads, it uses funding spreads of the processing organization. As noted before, in this thesis the borrowing and the lending rates are assumed to be the same. The FVA amounts can be seen in Table 19.

Table 19: FBA and FCA for the three counterparties

<table>
<thead>
<tr>
<th>Term</th>
<th>Citi</th>
<th>Goldman Sachs</th>
<th>JP Morgan</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCA</td>
<td>$-859.10</td>
<td>$-744.00</td>
<td>$-925.10</td>
</tr>
<tr>
<td>FBA</td>
<td>$960.10</td>
<td>$1,046.00</td>
<td>$1,038.00</td>
</tr>
<tr>
<td>FVA</td>
<td>$101.00</td>
<td>$302.00</td>
<td>$112.90</td>
</tr>
</tbody>
</table>

MVA is calculated from FCA using the following approximation from (Gregory,
\[ MVA \approx 2FCA \sqrt{\frac{\tau_{IM}}{T} \Phi^{-1}(\alpha)} \]

where \( T \) is the maturity of the netting set and \( \tau_{IM}T \) is the assumed time horizon (MVA days from Table 22), \( \Phi^{-1}() \) is the inverse of the cumulative normal distribution function and \( \phi() \) is the density function of normal distribution. In this approximation MVA (as seen in Table 20) is quite close to FCA.

Table 20: MVA for the three counterparties

<table>
<thead>
<tr>
<th>TERM</th>
<th>Citi</th>
<th>Goldman Sachs</th>
<th>JP Morgan</th>
</tr>
</thead>
<tbody>
<tr>
<td>MVA</td>
<td>$-633.70</td>
<td>$-548.80</td>
<td>$-682.40</td>
</tr>
</tbody>
</table>

As already noted, this thesis assumes IMM regulatory framework, with \( \alpha = 1.4 \), however it is not too typical that banks with IMM approval would use the regulatory prescribed value. In the xVA library, KVA is approximated with the following formula\(^9\):

\[ KVA = -0.5(CC_{default} + CC_{CVA})\sqrt{MROC} \]

where \( CC_{default} \) is calculated using the advanced internal ratings based (IRB) methodology, and the stressed \( R \), \( CC_{CVA} \) is calculated based on the standardized approach and the definition of inputs are as in Table 21. The KVA amounts from the simulation can be found in Table 22.

Table 21: Inputs for KVA calculation

<table>
<thead>
<tr>
<th>INPUT</th>
<th>NAME</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M )</td>
<td>Effective maturity</td>
<td>The latest date when the contract may still be active.</td>
</tr>
<tr>
<td>( CC_{default} )</td>
<td>Default risk capital charge</td>
<td>Captures the default risk embedded in portfolios of financial derivatives.</td>
</tr>
<tr>
<td>( CC_{CVA} )</td>
<td>CVA risk capital charge</td>
<td>Covers the risk of mark-to-market losses on the expected counterparty risk.</td>
</tr>
</tbody>
</table>

\(^9\)For more details see (Gregory, 2015, p.168) and (Basel Committee on Banking Supervisions, 2011)
Table 22: KVA amounts for the three counterparties with IMM

<table>
<thead>
<tr>
<th>Term</th>
<th>Citi</th>
<th>Goldman Sachs</th>
<th>JP Morgan</th>
</tr>
</thead>
<tbody>
<tr>
<td>KVA</td>
<td>$-8.56</td>
<td>$-5.90</td>
<td>$-5.75</td>
</tr>
</tbody>
</table>

### 2.4 Analysis of the results

The results of the calculations in this thesis must be considered along with its limitations. These calculations only serve the purpose of demonstration, since a number of assumptions and simplifications were made and the netting set in scope only consists of two vanilla interest rate swaps, which is rarely the case in a real investment bank’s portfolio, which has more sophisticated valuation and risk models. It is important to note that the actual xVA calculation can be extremely difficult due to the size and complexity of the portfolio.

![xVA components for the three counterparts](image)

Figure 15: xVA components for the three counterparts

Table 23 shows the results of the xVA calculation for the given netting set of two IR swaps. In the table *Price* refers to the risk neutral price of the netting set, whereas *Value* refers to how much the netting set is worth for the processing organization. While value is different for every counterparty, the price is the same for all three. This confirms the so-called Value-To-Me concept of Ruiz (2015a).
Table 23: xVA components for the three counterparties

<table>
<thead>
<tr>
<th>TERM</th>
<th>CITI</th>
<th>GOLDMAN SACHS</th>
<th>JP MORGAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$462.33</td>
<td>$462.33</td>
<td>$462.33</td>
</tr>
<tr>
<td>CVA</td>
<td>$-520.10</td>
<td>$-400.20</td>
<td>$-356.20</td>
</tr>
<tr>
<td>DVA</td>
<td>$537.30</td>
<td>$584.90</td>
<td>$580.90</td>
</tr>
<tr>
<td>FCA</td>
<td>$-859.10</td>
<td>$-744.00</td>
<td>$-925.10</td>
</tr>
<tr>
<td>FBA</td>
<td>$960.10</td>
<td>$1 046.00</td>
<td>$1 038.00</td>
</tr>
<tr>
<td>KVA</td>
<td>$-8.56</td>
<td>$-5.90</td>
<td>$-5.75</td>
</tr>
<tr>
<td>MVA</td>
<td>$-633.70</td>
<td>$-548.80</td>
<td>$-682.40</td>
</tr>
<tr>
<td>xVA</td>
<td>$-524.06</td>
<td>$-68.00</td>
<td>$-350.55</td>
</tr>
<tr>
<td>Value</td>
<td>$-61.73</td>
<td>$394.34</td>
<td>$111.78</td>
</tr>
</tbody>
</table>

The MtM value of the netting set is aggregated with xVA terms, which are quite significant compared to the price of the netting set. For instance in case of Citi Group, the xVA amount is larger than the original price as seen in Figure 15, which means that the value becomes negative for the PO. This might seem very strange, that an originally long position can turn to short if all the related costs and benefits are included.

Here both CVA, DVA and symmetric FVA are included, however as stated before, the common market practice is to only include CVA along with symmetric FVA or include bCVA, but exclude FBA to avoid double counting the impact of the widening credit spread of the PO. (Gregory, 2015)

It is clear from Figure 15 that KVA is not significant compared to other xVA terms. This can be because the exposure profile of the netting set over the first year is very low compared to the following years and the IMM framework focuses on short-term exposure.

The approximation presented in the previous section based on Gregory (2015) implies that the absolute amount of the MVA is less than the FCA. On one hand MVA can be seen as a funding cost based on the worst-case move of a portfolio during a relatively short time-frame, on the other hand the FCA is an expected cost of funding during the whole lifespan of the portfolio.

The most significant term is FBA for all three scenarios, however this might not be valid in reality, given here it is assumed, that the funding spread of lending and borrowing rates are the same. Typically investment banks fund themselves
from customer deposits, wholesale money markets, private or public unsecured or unsecured borrowing. For instance it might not be valid to define the funding spread from bond yields if the bank funds itself from depots. In general, funding costs (and benefits) in derivatives portfolios can be broken down to the impact of undercollateralisation, non rehypothecation and segregation.

Overall the results of the xVA calculation suggest that under some circumstances xVA amounts can be quite significant therefore need to be carefully considered. However, in modeling one should avoid too complex and computationally intensive solutions, rather focus on consistency, stability while keeping regulatory and accounting rules in mind.
PART V:

CONCLUSION
In conclusion, this part gives a summary of the structure of the thesis and the relevant findings. This thesis presented the new era of derivatives markets, it outlined the xVA terms and the risk factors they account for and it showed the results of a simple implementation of xVA calculation of a netting set consisting of two plain vanilla interest rate swaps.

The main purpose of this thesis was to give an overview of xVA terms and compare the value of the same netting set with three different counterparts having different margin agreement and counterparty risk.

In the first part the research problem, motivation and objectives were stated along with an introduction to the post-crisis derivatives markets. The second part provided a high level overview of derivatives markets and practitioners focusing on investment banks. In terms of derivatives valuation it presented the traditional risk-neutral pricing framework in contrast with the recent so-called Value-To-Me concept including xVAs. The third part focused on introducing several risk factors and the relating xVA components in terms. It highlighted several considerations about xVA in general. The fourth part presented step-by-step a simple xVA implementation and provided a primer on the numerical results.

The optimal total xVA charge depends on counterparty credit risk, funding costs, capital requirements, and many other factors. Hence, the choice of hedge and collateral might differ depending on these issues. This thesis concludes that the value of a netting set can be different depending on the counterparty and their CSA agreement and other specifications and therefore confirms the need for xVA calculation. What is more, in some cases xVA amounts can be quite significant compared to the risk neutral price therefore they need to be considered and accounted for.

However popular research topic it is, the xVA field itself is quite recent, therefore there are still debates around certain xVA terms and how to model them. In my view it will take some time to have clear and concise terminology along with best practices from the market. I would argue that the most challenging problem in the xVA world is to set up the modeling infrastructure, which is robust enough without being too costly to integrate into decision making system. A proper xVA modeling framework should be able to achieve such estimates at a high level of precision without significantly increasing the computation time. However regarding investment banks another great challenge in terms of adopting xVA will be to modify and adjust processes, organizational structures and control functions.

The field of xVA can be considered for an interdisciplinary research project as experience in computer science and IT infrastructure would be required. On the other hand it can be interesting from management, human resources, process engineering
and accounting perspective, as well.

All in all, a trend of moving towards standardized vanilla trades can be identified, given exotic derivatives are getting more and more costly to trade, but the landscape of derivatives markets is continuously changing due to regulatory pressure, accounting rules and unexpected events like the 2007-2008 crisis. In my opinion the growing xVA desks in banks and central counterparty clearing houses will have essential role in the derivatives trading business.
References


APPENDIX

R code

# Calibration of the one factor Hull White Model#

install.packages("drat")
install.packages("RQuantLib", type="binary")
suppressPackageStartupMessages(library(RQuantLib))

packageurl <- "https://cran.r-project.org/src/contrib/Archive/ggplot2/
ggplot2_2.0.0.tar.gz"
install.packages(packageurl, repos=NULL, type="source")
install.packages("plotly")
install.packages("ggplot2")
library(ggplot2)
library(plotly)

params <- list(
  tradeDate=as.Date('2016-10-25'),
  settleDate=as.Date('2016-10-25'),
  startDate=as.Date('2016-10-25'),
  maturity=as.Date('2031-11-17'),
  payFixed=TRUE,
  dt=0.5,
  strike=0.02,
  method="HWAnalytic",
  interpWhat="zero",
  interpHow="spline")

tsQuotes <- list(d1w =0.41/100,
  d1m =0.416/100,
  d3m = 0.495/100,
  d6m = 0.558/100,
  d9m = 0.594/100,
  d1y = 0.623/100,
  s2y = 0.715/100,
  s3y =0.787/100,
  s5y =0.931/100,
  s10y =1.218/100)

swaptionMaturities <- c(1,2,5,7,10)
swapTenors <- c(1,2,5,7,10)
volMatrix <- matrix( c(0.3469,0.3891, 0.4235, 0.4103, 0.3915, 0.4149,0.4313, 0.4229, 0.4082, 0.3899, 0.432,0.421, 0.3896, 0.3754, 0.3629, 0.3916,0.3845, 0.362, 0.3531, 0.3434, 0.3472,0.3388, 0.3322, 0.3283, 0.3188),
 ncol=5, byrow=TRUE)
p <- plot_ly(x=swapTenors, y=swaptionMaturities, z=volMatrix, type="surface", mode="Viridis")

pricing <- BermudanSwaption(params, tsQuotes, swaptionMaturities, swapTenors, volMatrix)

summary(pricing)

tsQuotes <- list(d1w =0.41/100,
 d1m =0.416/100,
 d3m =0.495/100,
 d6m = 0.558/100,
 d9m = 0.594/100,
 d1y = 0.623/100,
 s2y = 0.715/100,
 s3y =0.787/100,
 s5y =0.931/100,
 s10y =1.218/100,
 s15y=1.3/100)

times <-c(1,2,5,7,10)
curves <- DiscountCurve(params, tsQuotes, times)
zero<-data.frame(curves$zerorates)

zerotable<-data.frame(curves$table)
q <-plot_ly(table, x = table [,1], y = table [,2],mode = 'lines')
layout(title = "Zero rates from Hull–White model",
 scene = list(
 xaxis = list(title = "Date"),
 yaxis = list(title = "Zero rate")))

times <-seq(0,9.5,0.5)
times
curves <- DiscountCurve(params, tsQuotes, times)
curves
90 zero <- data.frame(curves$zerorates)
91 zero
92 table <- data.frame(curves$table)
93 table
94 r <- plot_ly(table, x = table[, 1], y = table[, 2], mode = 'lines')
95 layout(title = "Zero rates from Hull–White model",
96 scene = list(
97 xaxis = list(title = "Date"),
98 yaxis = list(title = "Zero rate")))
99
dcurve <- data.frame(seq(0, 9.5, 0.5), curves$discounts)
100 dcurve
dc <- plot_ly(dcurve, x = dcurve[, 1], y = dcurve[, 2], mode = 'lines')
101 layout(title = "Discount curve from Hull–White model",
102 scene = list(
103 xaxis = list(title = "Date"),
104 yaxis = list(title = "Discount curve")))
105
# Calibration of the CIR model based on (Fang et al., 2012)#
106
library(stats)
107 install.packages("cubature")
108 library(cubature)

109 a <- function(t, s, theta.vector, vector){
110 kappa <- theta[1]
111 cc <- theta[2]
112 sigma <- theta[3]
113 lambda0 <- theta[4]
114 gamma <- 0.5 * sqrt(kappa^2 + 2 * sigma^2)
115 a <- (2 * (kappa * cc) / (sigma^2)) * log((gamma * exp(0.5 * kappa * (s - t))) / (gamma * cosh(gamma * (s - t)) + 0.5 * kappa * sinh(gamma * (s - t))))
116 return(a)
117 }
118
119 b <- function(t, s, theta.vector){
120 kappa <- theta[1]
121 cc <- theta[2]
122 sigma <- theta[3]
123 lambda0 <- theta[4]
124 gamma <- 0.5 * sqrt(kappa^2 + 2 * sigma^2)
125 b <- (-sinh(gamma * (s - t))) / (gamma * cosh(gamma * (s - t)) + 0.5 * kappa * sinh(gamma * (s - t)))
126 return(b)
127 }
128 argument <- function(theta.vector, T.vector, r.vector, s0.vector, lgd){

67
kappa <- theta[1]
cc <- theta[2]
sigma <- theta[3]
lambda0 <- theta[4]
r0 <- vector()
v0 <- vector()
y <- 0
for (i in length(T)) {
  rr <- r[i]
  TT <- T[i]
  intr0 <- function(s) \{exp(-rr*s)*(1-exp(a(0,s,theta)+b(0,s,theta)*lambda0))\}
  inti <- adaptIntegrate(intr0, lower=0, upper=TT)$integral
  r0[i] <- exp(rr*TT)*(1-exp(a(0,TT,theta)+b(0,TT,theta)*lambda0)) + rr*inti
  intv0 <- function(s) \{exp(-rr*s)*exp(a(0,s,theta)+b(0,s,theta)*lambda0))\}
  intike <- adaptIntegrate(intv0, lower=0, upper=TT)
  v0[i] <- intike$integral
  y <- y + (s0[i] + v0[i] - lgd * r0[i])^2
}
return (y)

r <- as.vector(c(0.006283444, 0.007093764, 0.009285943, 0.010658968, 0.012215231)) # zero rates from HullWhite model
T <- as.vector(c(1, 2, 5, 7, 10))
lambda0 <- 0.001
lgd <- 0.6
s <- 0
s0 <- as.vector(c(0.26/100, 0.402/100, 0.771/100, 1.054/100, 1.224/100))
kappa_MS <- 0.3
c_MS <- 0.03
sigma_MS <- 0.1
theta <- as.vector(c(kappa_MS, c_MS, sigma_MS, lambda0))
MS <- argument(theta, T, r, s0, lgd)
param_MS <- theta
s0 <- as.vector(c(0.26/100, 0.402/100, 0.771/100, 1.054/100, 1.224/100))
kappa_citi <- 1.4
c_citi <- 0.02
sigma_citi <- 0.15
theta <- as.vector(c(kappa_citi, c_citi, sigma_citi, lambda0))
cit <- argument(theta, T, r, s0, lgl)
param_citi <- theta

s0 <- as.vector(c(0.373/100, 0.484/100, 0.904/100, 1.202/100, 1.425/100))
kappa_gm <- 1.2
c_gm <- 0.018
sigma_gm <- 0.22
theta <- as.vector(c(kappa_gm, c_gm, sigma_gm, lambda0))
gm <- argument(theta, T, r, s0, lgl)
param_gm <- theta

s0 <- as.vector(c(0.266/100, 0.348/100, 0.613/100, 0.841/100, 1.045/100))
kappa_jp <- 0.3
c_jp <- 0.02
sigma_jp <- 0.005
theta <- as.vector(c(kappa_jp, c_jp, sigma_jp, lambda0))
jp <- argument(theta, T, r, s0, lgl)
param_jp <- theta

T <- as.vector(c(1, 2, 3, 5, 10))
s0 <- as.vector(c(115.7/10^4, 222.5/10^4, 137.3/10^4, 270.9/10^4, 270.2/10^4))
kappa_f <- 0.150
c_f <- 0.050
sigma_f <- 0.050
theta <- as.vector(c(kappa_f, c_f, sigma_f, lambda0))
f <- argument(theta, T, r, s0, lgl)
param_f <- theta
param_f

fitting_errors <- c(MS, citi, gm, jp, f)
fitting_errors

param_list <- cbind(param_MS, param_citi, param_gm, param_jp, param_f)
param_list <- data.frame(param_list)
row.names(param_list) <- c("kappa", "c", "sigma", "lambda0")
param_list

x <- vector()
for (i in 1:4) {
  if (2 * param_list[1, i] * param_list[2, i] < param_list[3, i]^2) {
    x[i] <- colnames(param_list)[i]
  }
}

summary <- rbind(param_list, fitting_errors)
row.names(summary) <- c("kappa", "cc", "sigma", "lambda0", "calb.err")
err <- format(summary[5,], digits = 3, scientific = TRUE)
param_list <- format(summary[1:4,], digits = 3, scientific = FALSE)
rbind(param_list, err)

# Calculating the unconditional probability of default#
library(stats)
install.packages("sde")
library(sde)
library(ggplot2)
library(reshape2)

a <- function(t, s, theta.vector){
kappa <- theta[1]
cc <- theta[2]
sigma <- theta[3]
lambda0 <- theta[4]
gamma <- 0.5*sqrt(kappa^2+2*sigma^2)
a <- (-2*(kappa*cc)/(sigma^2))*log((gamma*exp(0.5*kappa*(s-t)))/(gamma*cosh(gamma*(s-t)) + 0.5*kappa*sinh(gamma*(s-t)))))
return(a)
}
b <- function(t, s, theta.vector){
kappa <- theta[1]
cc <- theta[2]
sigma <- theta[3]
lambda0 <- theta[4]
gamma <- 0.5*sqrt(kappa^2+2*sigma^2)
b <- (-sinh(gamma*(s-t)))/(gamma*cosh(gamma*(s-t)) + 0.5*kappa*sinh(gamma*(s-t)))
return(b)
}
lambda <- function(theta.vector, t, dt){
lambda0 <- theta[4]
if(t==0){
return(lambda0)
}
else {
kappa <- theta[1]
cc <- theta[2]
sigma <- theta[3]
M <- t/dt
lambda <- apply(sde.sim(X0 = lambda0, delta = dt, T = t, N = M, M

70
\[ \text{theta} = c ( \text{kappa} \ast c, \text{kappa}, \text{sigma}), \text{model} = "CIR" \]

\[ \text{lambda[M+1]} \]

calcPD <- function(theta.vector, T.vector) {
  kappa <- theta[1]
  cc <- theta[2]
  sigma <- theta[3]
  lambda0 <- theta[4]
  CondSurv <- vector()
  Surv <- vector()
  CumPD <- vector()
  PD <- vector()
  s <- 0
  lambda0 <- vector()
  # 0 - 1
  lambda[1] <- suppressMessages(lambda(theta, T[1], dt))
  CondSurv[1] <- exp(a(T[1], T[2], theta) + b(T[1], T[2], theta) * lambda[1])
  Surv[1] <- CondSurv[1]
  n <- length(T) - 1
  CumPD[1] <- 1 - Surv[1]
  for (i in 2:n) {
    t <- T[i - 1]
    s <- T[i]
    dt <- T[i] - T[1]
    lambda[i] <- suppressMessages(lambda(theta, T[i], dt))
    CondSurv[i] <- exp(a(t, s, theta) + b(t, s, theta) * lambda[i])
    Surv[i] <- Surv[i - 1] * CondSurv[i]
    CumPD[i] <- 1 - Surv[i]
    PD[i] <- CumPD[i] - PD[i - 1]
    table <- suppressWarnings(data.frame(cbind(T[1:(n)], lambda, CondSurv, Surv, CumPD, PD)))
  }
  return (table)
}

T <- seq(0, 10, 0.5)
lambda0 <- 0.001
kappa <- 0.300
c <- 0.030
\[ \sigma < -0.100 \]
\[ \theta < c(\kappa, c, \sigma, \lambda_0) \]
\[ \text{PD}_{\text{MS}} \leftarrow \text{calcPD}(\theta, T) \]
\[ \text{PD}_{\text{MS}} \]
\[ \kappa < -1.400 \]
\[ c < -0.020 \]
\[ \sigma < -0.150 \]
\[ \theta < c(\kappa, c, \sigma, \lambda_0) \]
\[ \text{PD}_{\text{Citi}} \leftarrow \text{calcPD}(\theta, T) \]
\[ \text{PD}_{\text{Citi}} \]
\[ \kappa < -1.2 \]
\[ c < -0.018 \]
\[ \sigma < -0.22 \]
\[ \theta < c(\kappa, c, \sigma, \lambda_0) \]
\[ \text{PD}_{\text{Goldman}} \leftarrow \text{calcPD}(\theta, T) \]
\[ \text{PD}_{\text{Goldman}} \]
\[ \kappa < -0.300 \]
\[ c < -0.020 \]
\[ \sigma < -0.005 \]
\[ \theta < c(\kappa, c, \sigma, \lambda_0) \]
\[ \text{PD}_{\text{JP Morgan}} \leftarrow \text{calcPD}(\theta, T) \]
\[ \text{PD}_{\text{JP Morgan}} \]
\[ \kappa < -0.1500 \]
\[ c < -0.050 \]
\[ \sigma < -0.0500 \]
\[ \theta < c(\kappa, c, \sigma, \lambda_0) \]
\[ \text{PD}_{\text{Funding}} \leftarrow \text{calcPD}(\theta, T) \]
\[ \text{PD}_{\text{Funding}} \]

\[ \text{PD}_{\text{list}} \leftarrow \text{cbind}(\text{PD}_{\text{MS}}PD, \text{PD}_{\text{Citi}}PD, \text{PD}_{\text{Goldman}}PD, \text{PD}_{\text{JP Morgan}}PD, \text{PD}_{\text{Funding}}PD) \]
\[ \text{PD}_{\text{list}} \leftarrow \text{data.frame}(\text{PD}_{\text{list}}) \]
\[ \text{row.names}(\text{PD}_{\text{list}}) \leftarrow \text{seq}(0, 9.5, 0.5) \]
\[ \text{colnames}(\text{PD}_{\text{list}}) \leftarrow \text{c}("\text{PD}_{\text{MS}}", "\text{PD}_{\text{Citi}}", "\text{PD}_{\text{Goldman}}", "\text{PD}_{\text{JP Morgan}}", "\text{Funding}" ) \]
\[ \text{PD}_{\text{list}} \]

\[ x \leftarrow \text{seq}(0, 9.5, 0.5) \]
\[ \text{cumPD}_{\text{list}} \leftarrow \text{data.frame}(x, \text{MS} = \text{PD}_{\text{MS}}CumPD, \text{Citi} = \text{PD}_{\text{Citi}}CumPD, \text{Goldman} = \text{PD}_{\text{Goldman}}CumPD, \text{JP Morgan} = \text{PD}_{\text{JP Morgan}}CumPD) \]
\[ \text{df} \leftarrow \text{melt}(\text{data} = \text{cumPD}_{\text{list}}, \text{id.vars} = "x") \]
p0<-ggplot(data = df, aes(x=x, y = value, colour = variable)) + geom_line(size=1)

p1<-p0 + theme(
  panel.background = element_rect(fill = "white")
)+ theme(aspect.ratio=0.5)+theme(legend.text = element_text(colour="black", size = 16))+ theme(legend.title=element_blank())

p1<-p1+theme(panel.background = element_rect(fill = "white", color = "black", size =0.5))

p2<-p1+theme(axis.text.x = element_text(
  colour = 'black', size = 14,
  hjust = 0.5, vjust = 0.5),axis.title.x=element_blank()) +
ylab("Cum PD")

p2<-p2+theme(axis.text.y=element_text(
  colour = 'black', size = 14,
  hjust = 0.5, vjust = 0.5))

#Calculate exposure metrics and xVA#

install.packages("xVA")
library(xVA)
library(ggplot2)
library(reshape2)

GenerateTimeGrid = function(col, maturity)
{
  remargin_freq_annum = col$remargin_freq/360
  mpor_days_annum = col$mpor_days/360
  time_points = seq(0, maturity, remargin_freq_annum)
  time_points_lookback = seq(remargin_freq_annum-mpor_days_annum,
                              maturity-mpor_days_annum,remargin_freq_annum)
  time_points = sort(c(time_points_lookback,time_points))
  return(time_points)
}

exposure = function(discount_factors, time_points, spot_curve, col,
                      trades, sim_data)
{
  num_of_points = length(time_points)
  num_of_trades = length(trades)
  Swap_MtMs    = matrix(0,nrow=sim_data$num_of_sims, ncol = num_of_points)
  Swap_MtMs_coll = matrix(0,nrow=sim_data$num_of_sims, ncol = num_of_points)
timesteps_diff = diff(time_points)

spot_interest_rate = spot_curve[1]
forward_curve = (discount_factors[1:(num_of_points−1)])/discount_factors[2:num_of_points−1]/timesteps_diff
forward_curve = c(spot_interest_rate,forward_curve)
forward_diff = diff(forward_curve)
theta = forward_diff + sim_data$mean_reversion_a*forward_curve[2:length(forward_curve)]
set.seed(30269)
interest_rates = rep(0,length(num_of_points))
interest_rates[1] = spot_interest_rate
random_numbers = matrix(runif(sim_data$num_of_sims*(num_of_points−1)),nrow=sim_data$num_of_sims,ncol=num_of_points−1)

for(index in 1:num_of_trades)
{
    maturity = trades[[index]]$Ei
    swap_rate = trades[[index]]$swap_rate
    BuySell = ifelse(trades[[index]]$BuySell=='Buy',1,−1)

time_points_temp = time_points[time_points<=maturity]
num_of_points_temp = length(time_points_temp)
A = rep(0,length(time_points_temp))
B = (1−exp(−sim_data$mean_reversion_a*(maturity−time_points_temp)))/sim_data$mean_reversion_a

disc_factors = matrix(0,nrow=num_of_points_temp,ncol=num_of_points_temp)
dt = maturity/num_of_points_temp

for(j in 1:sim_data$num_of_sims)
{
    Floating_leg = rep(0,num_of_points_temp)
    for(i in 2:num_of_points_temp)
    {
        interest_rates[i] = interest_rates[i−1] + (theta[i−1]−sim_data$mean_reversion_a*interest_rates[i−1])∗timesteps_diff[i−1]+ sim_data$volatility*qnorm(random_numbers[j,i−1])*sqrt(timesteps différence[i−1])
    }

    for (i in 1:num_of_points_temp)
    {
    }

}
for (i in 1:num_of_points_temp)
    disc_factors[i,1:(num_of_points_temp-i+1)] = A[num_of_points_temp-i+1]*exp(-B[num_of_points_temp-i+1]*interest_rates[1:(num_of_points_temp-i+1)])

for (i in 0:(num_of_points_temp-1))
    Floating_leg[i+1] = 1-disc_factors[num_of_points_temp-i,i+1]

Fixed_leg = dt*swap_rate*colSums(disc_factors, na.rm = TRUE)

Swap_MtM = (Floating_leg - Fixed_leg)*BuySell

Swap_MtMs[j,1:num_of_points_temp] = Swap_MtMs[j,1:num_of_points_temp] + Swap_MtM
Swap_MtMs_coll[j,1:num_of_points_temp] = Swap_MtMs_coll[j,1:num_of_points_temp] + col$ApplyThres(Swap_MtM)

exposure_profile = list()

exposure_profile$EE_uncoll = apply(apply(Swap_MtMs,c(1,2),function(x ) ifelse(x>=0,x,0)),2,mean,na.rm=TRUE)

exposure_profile$NEE_uncoll = apply(apply(Swap_MtMs,c(1,2),function(x ) ifelse(x<0,x,0)),2,mean,na.rm=TRUE)

exposure_profile$EE_uncoll[is.na(exposure_profile$EE_uncoll)] = 0

exposure_profile$NEE_uncoll[is.na(exposure_profile$NEE_uncoll)] = 0

exposure_profile$PFE_uncoll = apply(apply(Swap_MtMs_coll,c(1,2),function(x ) ifelse(x>0,x,0)),2,quantile,sim_data$PFE_Percentile,na.rm=TRUE)

exposure_profile$PFE_uncoll[is.na(exposure_profile$PFE_uncoll)] = 0

exposure_profile$EE = apply(apply(Swap_MtMs_coll,c(1,2),function(x ) ifelse(x>=0,x,0)),2,mean,na.rm=TRUE)

exposure_profile$EE[is.na(exposure_profile$EE)] = 0

exposure_profile$NEE = apply(apply(Swap_MtMs_coll,c(1,2),function(x ) ifelse(x<0,x,0)),2,mean,na.rm=TRUE)

exposure_profile$NEE[is.na(exposure_profile$NEE)] = 0

exposure_profile$PFE = apply(apply(Swap_MtMs_coll,c(1,2),function(x ) ifelse(x>0,x,0)),2,quantile,sim_data$PFE_Percentile,na.rm=TRUE)

exposure_profile$PFE[is.na(exposure_profile$PFE)] = 0

EEE = rep(0,num_of_points_temp)
EEE[1] = exposure_profile$EE[1]

for(i in 2:(num_of_points))
    EEE[i] = max(EEE[i-1], exposure_profile$EE[i])

exposure_profile$EEE = EEE

return(exposure_profile)

raw_data <- read.table("Spot_Curve.csv", header=TRUE, sep=";", dec=".")
raw_data <- data.frame(raw_data[1:20,])
Tenors <<- as.numeric(raw_data[,1])
Rates <<- as.numeric(raw_data[,2])
interp.function <<- splinefun(Tenors, Rates, method="natural")

swap1 = IRSwap(Notional=1, Currency="USD", Si=0, Ei=10, BuySell='Buy', swap_rate=0.005)
swap2 = IRSwap(Notional=1, Currency="USD", Si=0, Ei=4, BuySell='Sell', swap_rate=0.003)
trades <- list(swap1, swap2)

sim_data = list(PFE_Percentile = 0.9, num_of_sims = 1000,
                 mean_reversion.a = 0.0633, volatility = 0.0239)

col_Citi = CSAb(thres_cpty = 0.0002, thres_PO = 0.0001, IM_cpty = 0.015,
                IM_PO = 0.01, MTA_cpty = 0.004, MTA_PO = 0.005, mpor_days = 10,
                remargin_freq = 20)
maturity <- max(as.numeric(lapply(trades, function(x) x$Ei)))
time_points_Citi = GenerateTimeGrid(col_Citi, maturity)
spot_curve_citi=c(interp.function(time_points_Citi))
discount_factors_Citi = exp(-time_points_Citi*spot_curve_citi)
exposure_Citi <- exposure(discount_factors_Citi, time_points_Citi, spot_curve_citi, col_Citi, trades,
                           sim_data)

col_CM = CSAb(thres_cpty = 0.0001, thres_PO = 0.0001, IM_cpty = 0.0130,
               IM_PO = 0.01, MTA_cpty = 0.004, MTA_PO = 0.005, mpor_days = 10,
               remargin_freq = 20)
maturity <- max(as.numeric(lapply(trades, function(x) x$Ei)))
time_points_CM = GenerateTimeGrid(col_CM, maturity)
spot_curve_GM=c(interp_function(time_points_GM))
discount_factors_GM = exp(-time_points_GM*spot_curve_GM)
exposure_GM<exposure(discount_factors_GM, time_points_GM, spot_curve_GM, col=col_GM, trades, sim_data)
col_JP = CSAb(thres_cpty = 0.0001, thres.PO = 0.0001, IM_cpty = 0.012,
IM.PO = 0.01, MTA_cpty = 0.004, MTA.PO =0.005, mpor_days = 5,
remargin_freq = 20)
maturity <- max(as.numeric(lapply(trades, function(x) x$Ei)))
time_points_JP = GenerateTimeGrid(col_JP, maturity)
spot_curve_JP=c(interp_function(time_points_JP))
discount_factors_JP = exp(-time_points_JP*spot_curve_JP)
x<time_points_Citi
Citi EE_list <- data.frame(x, EE_uncoll=exposure_Citi$EE_uncoll)
EE_Citi <- melt(data = Citi EE_list, id.vars = "x")
p1<-ggplot(data = EE_Citi, aes(x=x, y = value*10000, colour = variable)) + geom_line(size=1)
p1<-p1+theme(panel.background = element_rect(fill = "white", color =" black", size =2))
p1<-p1+theme(aspect.ratio=0.3)
p1<-p1+theme(legend.text = element_text(colour="black", size = 16))
p1<-p1+theme(legend.title=element_blank())
p1<-p1 +theme(axis.title.x = element_text(
colour = 'black', angle = 90, size = 13,
hjust = 0.5, vjust = 0.5),axis.title.x=element_blank()) +
ylab("Expected Exposure")
p1

data <- read.table("PD.csv", header=TRUE, sep=";", dec=".")
PD<--data.frame(data)
colnames(PD)<-c("PD_MS","PD_Citi","PD_Goldman","PD_JPMorgan","PD_Funding")
Rates <- as.numeric(PD$PD_MS)
interp_function <- splinefun(Tenors, Rates, method="natural")
PD_PO_Citi=c(interp_function(time_points_Citi))
Rates <- as.numeric(PD$PD_MS)
interp_function <- splinefun(Tenors, Rates, method="natural")
PD_P0_GM = c(interp_function(time_points_GM))

Rates = as.numeric(PDSPD_MS)
interp_function = splinefun(Tenors, Rates, method="natural")
PD_P0_JP = c(interp_function(time_points_JP))

Rates = as.numeric(PDSPD_Citi)
interp_function = splinefun(Tenors, Rates, method="natural")
PD_Citi = c(interp_function(time_points_Citi))

Rates = as.numeric(PDSPD_Goldman)
interp_function = splinefun(Tenors, Rates, method="natural")
PD_GM = c(interp_function(time_points_GM))

Rates = as.numeric(PDSPD_JPMorgan)
interp_function = splinefun(Tenors, Rates, method="natural")
PD_JP = c(interp_function(time_points_JP))

cpty_LGD = 0.6
PO_LGD = 0.6

reg_data_Citi = list(framework="IMM", PD = 0.09, LGD = 0.6, return_on_capital = 0.0337, cpty_rating = 'A', mva_days = 10, mva_percentile = 0.99)
reg_data_GM = list(framework="IMM", PD = 0.008, LGD = 0.6, return_on_capital = 0.0337, cpty_rating = 'A', mva_days = 10, mva_percentile = 0.99)
reg_data_JP = list(framework="IMM", PD = 0.007, LGD = 0.6, return_on_capital = 0.0337, cpty_rating = 'A', mva_days = 10, mva_percentile = 0.99)

reg_data_Citi = list(framework="IMM", PD = 0.09, LGD = 0.6, return_on_capital = 0.0337, cpty_rating = 'A', mva_days = 10, mva_percentile = 0.99)

PD_FVA_Citi = c(interp_function(time_points_Citi))
PD_FVA_GM = c(interp_function(time_points_GM))
PD_FVA_JP = c(interp_function(time_points_JP))

xVA_Citi = list()
xVA_Citi$KVA = calcKVA(exposure_Citi, col_Citi, trades, reg_data_Citi, time_points_Citi)
xVA_Citi$CVA = CalcVA(exposure_Citi$EE, discount_factors_Citi, PD_Citi, cpty_LGD)
xVA_Citi$DVA = CalcVA(exposure_Citi$NEE, discount_factors_Citi, PD_P0_Citi, PO_LGD)
xVA_Citi$FCA = CalcVA(exposure_Citi$EE, discount_factors_Citi,
PD_FVA_Citi

xVA_Citi$FBA = CalcVA(exposure_Citi$NEE, discount_factors_Citi, PD_FVA_Citi)
xVA_Citi$MVA = xVA_Citi$FCA*2*sqrt(reg_data_Citi$mva_days/(250*maturity))*qnorm(reg_data_Citi$mva_percentile)/dnorm(0)

xVA_Citi

xVA_GM = list()
xVA_GM$KVA = calcKVA(exposure_GM, col_GM, trades, reg_data_GM, time_points_GM)
xVA_GM$CVA = CalcVA(exposure_GM$SEE, discount_factors_GM, PD_GM, cpty_LGD)
xVA_GM$DVA = CalcVA(exposure_GM$NEE, discount_factors_GM, PD_PO_GM, PO_LGD)
xVA_GM$FCA = CalcVA(exposure_GM$SEE, discount_factors_GM, PD_FVA_GM)
xVA_GM$FBA = CalcVA(exposure_GM$NEE, discount_factors_GM, PD_FVA_GM)
xVA_GM$MVA = xVA_GM$FCA*2*sqrt(reg_data_GM$mva_days/(250*maturity))*qnorm(reg_data_GM$mva_percentile)/dnorm(0)

xVA_GM

xVA_JP = list()
xVA_JP$DVA = CalcVA(exposure_JP$NEE, discount_factors_JP, PD_PO_JP, PO_LGD)
xVA_JP$MVA = xVA_JP$FCA*2*sqrt(reg_data_JP$mva_days/(250*maturity))*qnorm(reg_data_JP$mva_percentile)/dnorm(0)

xVA_JP

xVA<-data.frame(cbind(Citi=xVA_Citi, Goldman=xVA_GM, JPMorgan=xVA_JP))