Global Stock Market Volatility Connectedness

A comprehensive analysis of volatility connectedness of global stock market indices

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„Én, Kapronczay Mór teljes felelősségem tudatában kijelentem, hogy a jelen szakdolgozatban szereplő minden szövegrész, ábra és táblázat - az előírt szabályoknak megfelelően hivatkozott részek kivételével - eredeti és kizárólag a saját munkám eredménye, más dokumentumra vagy közreműködőre nem támaszkodik."
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1. Introduction

Volatility has always been in the centre of interest concerning financial markets. Practically speaking, volatility is the measure of the variability of the price of a financial instrument. Consequently, volatility is also a measure of risk for an investor. No wonder that volatility trading intensified to such an extent, that their sheer presence became a factor that drives financial markets at least on the short run (Pisani, 2018).

During time of crises volatility of financial instruments tend to increase (Manda, 2010), which is a common knowledge by now. It is also a widely recognized fact that correlation between returns of financial instruments tends to increase in turbulent times. The spread of a crisis is often documented as contagion, as if some financial asset contaminated others with high volatility (and often decreasing prices). This phenomenon is in the centre of my paper. In my thesis I only examine volatility of financial instruments, and exclude returns as a metric of examining financial instrument connectedness. This is a choice made to ensure narrowing, therefore ensuring deep coverage of the topic, and because of the recently mentioned intensification of volatility trading.

The relevance of my analysis is given by the aforementioned well-known facts. How volatility is spreading from one market to another? What proportion of volatility is attributable to systematic or idiosyncratic reasons? During crises, when volatility increases, and correlation between returns increases as well is because of the rising global or regional factor, or is it a consequence of volatility spillover stemming from local causes? If both, to what extent each is responsible? Finding the answers to these questions is a hard task, and the main value of my thesis is the analysis of local volatility time series thereby dividing the impact of global, regional and local factors. This is extremely relevant in terms of diversification strategy: if I am to prepare for a crisis, how shall I split my investments between stocks in the world’s financial markets?

Volatility of financial markets can be represented through volatility of stock market indices. These indices summarize some or all the most important stocks traded on the stock exchange with a weighting based mostly on the market capitalization of companies. Thereby one can get insight about the whole market’s volatility by looking at one number, which aggregates stock volatilities and weights them according to their importance.
In my thesis I will use the Diebold-Yilmaz Spillover methodology on volatility time series of 16 leading stock market indices, which are from Europe, North America, South America and Asia as well, thereby representing the global financial markets. From each continent 4 important stock indices were chosen.1 From these volatility time series, a global volatility factor, and regional volatility factors are extracted, thereby giving the opportunity to separately examine the effect of global and regional factors, the role of global, regional and that of idiosyncratic shocks. Consequently, the Diebold-Yilmaz Spillover methodology will at the same time be used on volatility time series, and on local volatility time series, from which the global and regional factors had been previously extracted. In this sense, my motivation in extracting the global factor is similar to that of Byrne et al.’s (2015). Using the Diebold-Yilmaz Spillover Framework creates the opportunity to examine the spread of volatility from one index to another. An extremely important attribute of this methodology is it can show directional relations between the set of time series, and also produces comparable (normalized) values of connectedness.

It is important to note however, that interpreting volatility as standard deviation of returns can be misleading. Volatility is a latent process that cannot be observed directly, but can only be estimated (Molnár, 2012, p. 20). In my paper, I will show how volatility can be estimated. The first important choice is whether one uses high frequency data (i.e. 5-minute returns), or estimates volatility from daily open, close, high and low prices, with a range based estimator. I chose the latter because according to Molnár (2012), they provide a sufficiently accurate estimation of realized volatility.

State-space models are important tools for my analysis. This methodology stems from engineering sciences, and had been used for tracking the exact position of the Voyager spacecraft, for example. A new momentum in the popularity of these models came when Rudolf Kálmán introduced the Kalman-filter (Kalman, 1960) for fitting and smoothing these models on data. (Campagnoli et al., 2007)

The most important rationale in using state-space models is that it enables us to estimate latent factors from noisy observations (Campagnoli et al., 2007). And this is not just about volatility being latent, but that stock market volatility can arise from different reasons. On the one hand, there must be global reasons, which affect all stock exchanges, and idiosyncratic shocks affecting one, or regional shocks affecting a set of stock exchanges. To achieve separation of

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1 This is further discussed in Section 3 (Data).
volatility stemming from these different factors, a global, and some regional volatility factors can be estimated using state-space models. These factors, of course, are even indirectly unobservable, being an abstract construction, which can be extracted from the individual volatility time series using the above methodology.

It is also important to note, that financial markets are not separated from each other. Globalization is in one of its most advanced stages in terms of financial markets. Therefore, the spreading of volatility from one market to the other is a directly observable phenomenon. To quantify and reliably describe this spreading, Diebold & Yilmaz (2009) introduced the Diebold-Yilmaz Spillover Index, which uses vector-autoregressive models and variance decomposition to provide information about the direction and magnitude of spillover between time series.

The structure of my thesis is as follows. In Section 2, the methodology used in this paper is discussed thoroughly. Section 3 presents data used in this research. Then, in Section 4 the results of the conducted analysis are demonstrated. Results include the fitted state-space model, the global and regional volatility factors, spillover in terms of volatilities and local factors, and dynamic analysis of network connectedness and net regional spillover, namely the net volatility transmitting and receiving between regions. After that, in Section 5 some robustness checks are shown. Finally, Section 6 concludes and summarizes the relevant findings.
2. Methodology & theoretical background

In this section the methodology used in my thesis is presented. Firstly, the broad subject of stock market connectedness is covered. Secondly, a range-based volatility estimator is introduced. Thirdly, state-space models are discussed, in which the exact specification of the models used in this thesis are shown. And lastly, the reader is shown into the Diebold-Yilmaz Spillover Framework.

2.1. Stock market connectedness

Connectedness of financial instruments has been a topic of significant interest as their importance grew. Financial crises gave another crucial aspect on this topic: during crises the returns of financial instruments tend to increasingly move together empirically. This phenomenon is so broad it can be mentioned among the stylized facts shaping global financial markets.

In addition, as globalization prevails, and far points of the world got closer and closer to each other in terms of communication, news availability, etc. this phenomenon gets even stronger. On top of that, financialization is also a meaningful factor. This means that more and more complex financial assets create the opportunity to trade on virtually any information one has, while more and more raw materials are available on commodity exchanges, further expanding the opportunities of trading.

By this, one would expect the rise of connectedness of financial assets over time. But what does connectedness mean? Concerning this issue, many question arises. It is easy to suspect, that connectedness could be measured through the time series of the concerning financial assets. From a probability theory point of view, the variable of interest is price change, or return. The first moment of the distribution of this variable is the expected value of returns, while the second is return volatilities. Therefore, one can define connectedness based on some measure of returns or of return volatilities\(^2\).

In my thesis, I focus on volatility connectedness, and leave out returns as a measure of interest. Apart from the reasons mentioned in the Introduction, this is mainly because of what the two metrics represent. Return connectedness means that prices move the same direction, while volatility connectedness shows to what extent the prices move – in either direction. Volatility

\(^2\) Section 2.2 discusses the estimation of volatility.
is often quoted as a measure of investor fear. During crises, volatility tends to move up, while prices go down. Tracking how volatility increases or decreases in financial markets can provide information on how the increasing trading that come with crises spreads. This way, volatility connectedness is “fear connectedness” (Diebold & Yilmaz, 2014, p. 5), a measure of how investor fear starts to dominate the sentiment of financial markets.

One plausible measure of connectedness is of course correlation or covariance. Correlation, in a probability theory point of view is the expected value of the product of two centralised random variables. Baig & Goldfajn (1999) examines the spread of crisis in the Asian financial markets during the Asian crisis of 1997. Through return correlations the authors find increasing connectedness through the crisis in terms of currency and sovereign spreads, while the results are not so clear in terms of equity markets. Increasing correlation between financial assets during crises are found by Kenourgios et al., (2011) using a multivariate regime-switching Gaussian copula model and computing asymmetric generalized dynamic conditional correlation (AG-DCC). In their paper, emerging equity markets are examined. In the paper of Kenourgios & Dimitrou (2015), a similar analysis is conducted using different methodology, across regions and economic sectors. The results are very close to Kenourgios et al. (2011), providing evidence on the existence of increasing comovements in times of the Global Financial Crisis of 2007-2009.

This increasing correlation is often referred to as contagion, as it is supposed that some key players contaminate the market. To put it differently, there are important nodes in the network of financial markets, where somehow information appears, and spreads all over the world. The measurement of contagion is of course impossible using only correlations. This is a consequence of the two weaknesses of correlation: it does not have direction, and it can only show pairwise relations. To overcome these flaws, a more sophisticated methodology of the Diebold-Yilmaz Spillover Framework is employed in my thesis, which is described in Section 2.4.

However, there are two additional papers which are impossible to leave out from discussion of this topic. The earlier, written by King et al. (1994) examines stock market volatility connectedness in order to assess financial integration across time. They find that comovements depend on observable and unobservable factors, which determine the premium achievable investing in these instruments. The latter, by Forbes and Rigobon (2002) finds that there is a heteroskedasticity bias in correlation between financial asset returns. Namely, when volatility...
is higher, correlation is higher too. When one accounts for that bias, there is virtually no increase in correlation during the crises investigated. In my thesis, I aim to add to academic discussion of the topic of financial market integration.

2.2. Estimating volatility

My thesis utilises volatilities of stock indices. As it was mentioned above, “volatility is latent, therefore must be estimated” (Molnár, 2012, p. 20). Nowadays it is extremely popular to use realized volatility as an estimator. This practically means summarizing the squared returns of very short time periods. This approach is an unbiased estimator of volatility, the second moment of the distribution of returns. A large number of papers discusses how to optimally construct a realized volatility estimator, there is a tradition of using 5-minute returns (Andersen et al., 2011). Liu et al. (2015) showed in an extensive study that summarizing the squared 5-minute returns is a proper way of estimating volatility, because other more sophisticated procedures cannot significantly overperform the simple method.

However, in my paper a different approach is employed. Molnár (2012) deeply discusses the properties of range-based volatility estimators, where no high frequency intraday data is needed to estimate volatility. In this framework, volatility is the daily diffusion parameter of returns following a geometric Brownian motion (Molnár, 2012, p. 21.). To use all available information, parameters of a volatility estimator are according to Molnár (2012) the difference of the natural logarithm of close (c), high (h) and low (l) daily price to the natural logarithm of open price. Garman & Klass (1980) finds that the minimum variance analytical estimator can be further simplified to Equation 1. In this thesis this simplified version is used to estimate volatility.

\[
\sigma^2 = 0.5 \ast (h - l)^2 - (2 \ast ln(2) - 1) \ast c^2 \quad (Molnár, 2012, p. 22)
\]

2.3. State-space model

In writing Subsection 2.3 I am heavily relying on Commandeur & Koopman’s (2007) book, specifically on Section 9 in it. Using state-space models is useful when one assumes that the observed time series is driven by an unobservable process, which is referred to as the “state” of the system. The state can be a multidimensional object as well, in that case the state of the system is a multidimensional vector. This vector shows the state of the system in every time
period \( t \). The observed time series are supposed to be noisy observations of the state of the system.

State-space models can be formulated through a measurement and a state equation. The state equation identifies the dynamics of the state of the system through time. Therefore, this equation creates connection between the states of the system at different times. The state of the system in most of cases is driven by an autoregressive process, the states are somewhat persistent thereby with an error term. The extent of this persistence is a question of great interest generally. To achieve flexibility in fitting these models, a white noise process is present in the equation with expected value of 0 and variance of \( \sigma^2 \). Direct example will be shown in Subsection 2.3.1 and 2.3.2.

The measurement equation on the other hand connects the state of the system with the observed time series. The structure of these equations are as follows: the state of the system is transformed some way (a linear transformation is assumed throughout my thesis) and a realization of an above mentioned white noise process is added to it. Direct example will also be shown in Subsection 2.3.1. and 2.3.2. Nevertheless, these equations in the case of multivariate analysis can be presented in the form of matrix equations.

2.3.1. Global-Local model

In the first model an attempt to estimate the global volatility factor is made. Apart from that, there is the aim of identifying the connection between the latent global factor and volatilities. Subsequently, the latent global factor can be filtered from volatilities thereby getting the opportunity to compare the connection between volatilities and between the local factor of volatilities using the Diebold-Yilmaz framework. The Global-Local model used in this research gained inspiration from Diebold & Li’s (2006) work. In that paper, the authors use the Dynamic Nelson-Siegel (Nelson & Siegel, 1987) approach of modelling yield curves to forecast the United States government bonds’ yields. In this model, the yield curve of the US is constructed from 3 factors responsible for the short, medium and long-term yields. Betas for these factors shape the yield curve, and virtually every yield curve can be drawn with different betas. The authors are successful in forecasting the yield curves using this approach.

Diebold et al. (2008) are building on the previous model by extending it to a multi-country environment. In this research, every country has its own country factors and its own betas representing the sensitivity of the yield curve concerning the factor. These country factors
depend on a global factor with a country-specific beta, and the global factor has autoregressive dynamics. Morita & Bueno (2008) does approximately the same research with a slightly different methodology in estimating the model, and on yield curves of other countries on a different time-span.

A further sophistication of this methodology includes other factors than the global factor to the analysis. Bae & Kim (2011) introduces regional factors for continents as well. In my thesis, I firstly aim to estimate the global volatility factor by filtering it out from the above mentioned 16 volatility time series. Secondly, in the Global-Regional-Local model a more sophisticated model close to Bae & Kim’s (2011) is estimated. This setting is described in Subsection 2.3.2. In the Global-Local setting my assumptions are very close to Diebold et al.’s (2008) and Morita & Bueno’s (2008). The volatilities of stock indices depend on the global factor with a particular beta. The estimated state-, and measurement equations are as follows (Equation 2-3 respectively):

\[
(2) \quad X_t = A * X_{t-1} + B * u_t \\
(3) \quad y_t = C * X_t + D * \epsilon_t
\]

Where \(X_t\) is the state column vector describing the state of the system (17 elements for every time \(t\)), \(A\) is the state-transition matrix (17x17, constant in time), \(X_{t-1}\) is the state column vector one period before the current. \(B\) is the disturbance loading matrix (17x17, constant in time) which connects \(u_t\) into the system. \(u_t\) is a column vector (of 17 elements for every time \(t\)) containing error terms so that \(E(u_t * u_{t'}) = \sigma_i^2\) if \(t=t'\), otherwise 0. \(Y_t\) is a column vector of the observed volatilities (16 element for every time \(t\)). \(C\) is the measurement sensitivity matrix (16x17, constant in time) which is responsible for establishing connection between observations and the state of the system. \(D\) is the observation innovation matrix (16x16, constant in time) that channels \(\epsilon_t\) column vector of error terms in (16 elements for every time \(t\)). \(\epsilon_t\) is so that \(E(\epsilon_t * \epsilon_{t'}) = \sigma_i^2\) if \(t=t'\), otherwise 0. I assume that cross-effects are negligible, therefore only the diagonal elements of \(A\), \(B\), and \(D\), and the first column of \(C\) need to be optimised, other elements of matrices being 0 or 1.

To conclude, in this specification volatilities are assumed to follow the equation:

\[
(4) \quad vol_{i,t} = \beta_i * G_t + \epsilon_{i,t},
\]

where \(vol_{i,t}\) is stock index volatility \(i\) in time \(t\), \(\beta_i\) is the stock index’s sensitivity on the global factor \(G\), \(G_t\) is the factor’s value in time \(t\), and \(\epsilon_{i,t}\) is the above-mentioned error term.
My estimation strategy will consist of methods for finding the initial values for these values. This strategy is extremely important, because the estimation by the Kalman-filter “… is very sensitive to the initial values” (Morita & Bueno, 2008, p.10). These methods are “educated guesses” for finding values close enough to the expected real values in these matrices. Therefore, some less sophisticated processes are used that produce approximate values for the coefficients.

The importance of filtering out the part of volatilities attributable to global factors can be easily seen from a diversification point of view. Let’s suppose that there are two investment options for me, stock index A and B, and I am having a stake in low volatility. Let’s also suppose, that A is driven mainly by the global factor, while B being roughly independent from it. Knowing this, I can choose which option I invest my funds in, or the ratio between them. If I expect relatively quiet times in global stock exchanges I choose A, otherwise I reallocate my funds in the direction of B.

2.3.2. The Global-Regional-Local model

The model specification shown in Subsection 2.3.1 can be further sophisticated. Even though a global factor is extracted, there can be significant communalities between volatility time series. Another way of saying the same is that there can be other factors, apart from global ones, that drive volatilities that are not idiosyncratic factors. One possible assumption is that the remaining common factors are also geographically common: hence, the rationale of regional factors. This approach is further strengthened by empirical findings in Section 4.

In this specification, apart from the global factors 4 regional factors are estimated. These factors aim to represent continents: North-America, South-America, Europe and Asia. All of these factors are assumed to be independent from each other and follow autoregressive dynamics:

\[
G_t = \varphi_G * G_{t-1} + u_{G,t}
\]
\[
R_{NA,t} = \varphi_{NA} * R_{NA,t-1} + u_{NA,t}
\]
\[
R_{SA,t} = \varphi_{SA} * R_{SA,t-1} + u_{SA,t}
\]
\[
R_{EU,t} = \varphi_{EU} * R_{EU,t-1} + u_{EU,t}
\]
\[
R_{AS,t} = \varphi_{AS} * R_{AS,t-1} + u_{AS,t}
\]

Equation 5 shows the autoregressive equation of the global factor, while Equations 6-9 shows that of the regional factors. \(\varphi\) is the autoregressive coefficient of the factors, while \(u\) is an error.
term, with attributes specified before. Here I assumed that the idiosyncratic parts of volatilities follow a simple autoregressive process, too with $\tau_{i,t}$ being an error term having well-known attributes:

\begin{equation}
\epsilon_{i,t} = \theta_t \ast \epsilon_{i,t-1} + \tau_{i,t}
\end{equation}

Equation 5-10 are the state equations of the system, while the measurement equation can be easily formulated as a matrix equation. It is just like Equation 3, the difference being in matrix C. Here, C is a matrix (16x21) where the first 5 columns are organised in order to ensure that the global factor affects all volatilities, while the regional factors only affect the 4 corresponding volatilities. This is achieved through all elements being 0 in columns 2-5, except for the first 4 element in column 2, elements 5-8 in column 3, and so on.

Consequently, volatilities in this system follow Equation 11. $R_t$ in this equation refers to the corresponding regional factor, while $\beta_{R,i}$ is the sensitivity of $vol_i$ to the regional factor.

\begin{equation}
vol_{i,t} = \beta_{G,i} \ast G_t + \beta_{R,i} \ast R_t + \epsilon_{i,t}
\end{equation}

2.4. Diebold-Yilmaz spillover measure

The Diebold-Yilmaz Spillover Framework is a useful tool for measuring connectedness of time series. In the past, there were other approaches as well, the most known of them is correlation. Correlation is widely used to measure connectedness, but as it was mentioned before, can only catch pairwise relations and has no direction – both are enormous limitation in the context of financial markets.

Diebold & Yilmaz (2014) review different methodologies aiming to measure connectedness more effectively. As they say: “The equi-correlation approach of Engle and Kelly (2009), for example, uses average correlations across all pairs. The CoVaR approach of Adrian and Brunnermeier (2008) and the marginal expected shortfall approach of Acharya et al. (2010) track association between individual-firm and overall market movements and also rely less on linear Gaussian methods. Although these and various other measures are certainly of interest, they measure different things, and a unified framework remains elusive.” (p. 121)

Diebold & Yilmaz (2009) address that need by developing the Diebold-Yilmaz Spillover Framework. This approach focuses on variance decompositions associated with VAR models, by which it is able to distil directional information as well. The basis of the idea is simply “for each asset i we simply add the shares of its forecast error variance coming from shocks to asset
j, for all j≠i, and then we add across all i = 1, ..., N.” (pp. 158-159.) Therefore, this approach can measure the connectedness of the system and measure the direction and strength of connection between any subset of the analysed time series.

More precisely, consider an N-variable vector autoregressive model. A vector autoregressive model is a generalization of a 1-variable autoregressive model (AR), where dependency is not only possible in terms of past values of the time series, but even on past values of other time series. This dependency is the essence of the Diebold-Yilmaz framework. In Equation 12, $X_{t+1}$ is the vector of time series to be forecasted, $\theta * X_t$ is the Wiener-Kolmogorov linear least-squares forecast (Diebold-Yilmaz, 2009, p. 159.), $\epsilon_{t+1,t}$ is the linear one step-ahead error vector of particular interest.

\[
(12) \quad X_{t+1} = \theta * X_t + \epsilon_{t+1,t} \quad ; \quad X_{t+1,t} = \theta * X_t
\]

This forecast error $\epsilon_{t+1,t}$ is defined as the difference between the Wiener-Kolmogorov linear least-squares forecast and the actual $X_{t+1}$. Now for the sake of simplicity let us consider a two-variable VAR model. The variance of $\epsilon_{t+1,t}$ is a sum of forecast error variance of each variable of X, in this case $X_1$ and $X_2$. These forecast error variances of each variable can be decomposed to parts attributable to shocks in $X_1$ and $X_2$. Therefore, the forecast error variance can be decomposed to “own variance shares” (Diebold & Yilmaz, 2009, p. 159.) attributable to shocks in the same variable, and to “spillovers” (Diebold & Yilmaz, 2009, p. 159.) attributable to shocks in other variables.

However, the framework had its limitations. Most importantly, the framework relied on Cholesky-factor identification of VAR’s, therefore the results were dependent on the ordering of the variables. Diebold & Yilmaz (2012) further develop the framework by utilizing the generalized vector autoregressive framework. In this paper, the generalized framework is used, the calculations were made on Matlab2017a software.

To conclude the general discussion of the framework, one need to mention additional two important parameters in the framework. Namely, the order of the order of the underlying VAR-model, and the forecast horizon. First, the order of the VAR-model artlessly means how many lags of $X$ are included in forecasting $X_{t+1}$. On the other hand, forecast horizon (denoted with $H$) means how “far” ahead the forecast is made. A widely used $H=10$ setting would mean that forecast is made to $X_{t+1}, X_{t+2}, ... X_{t+9}$, and spillover are summarized throughout the time horizon.
Directional spillover can be quantified by the before variance decomposition, as a sum of all spillover. Directional spillover between time series can be formulated as: what part of the forecast error variance of \( X_i \) comes from shocks in \( X_j \). To summarise the volatility transmitting of \( X_j \), one has to sum up the before mentioned part for every \( i \neq j \). The same is true vica versa for all volatility absorption. By netting these values one can obtain net directional spillover, which quantifies how much an asset’s volatility is a leader (transmitting of volatility) or a follower (absorbing volatility) in financial markets.

Despite being just recently introduced, the Diebold Yilmaz framework had been used to wide variety of problems. Diebold & Yilmaz (2009) in a paper introducing the methodology analyses stock market connectedness in terms of returns and volatilities. Diebold & Yilmaz (2012) extends the methodology, and applies it to asset classes. Diebold & Yilmaz (2014) examines the connectedness of financial terms with emphasis on the 2007-08 financial crisis. Demirer et al (2017) discusses the global network of banks. Barunik et al. (2014) extends the analysis by separating good and bad volatility: the spillover from negative and positive returns, and discusses petroleum markets. Barunik et al (2016) continue with U.S. stock market. In the last 2 papers SAM, Spillover Asymmetry Measure is introduced, a tool for measuring the different characteristics in the spread of good (positive returns) and bad (negative returns) volatility.

### 2.4.1. Spillover table

The spillover table is a well-organized output of the above methodology. The table shows directional connections between all pair of time series. If one analyses \( n \) time series, then the table is going to be of \( nxn \) size. Element of row \( i \), and column \( j \) will show time series \( j \)’s effect on time series \( i \).

Total spillover in an N-variable VAR model can be obtained by subtracting own variance shares from the total forecast error variance. The value of total spillover is often normalized by dividing it with the total forecast error variance, thereby getting the ratio of spillover in total forecast error variance. This value is referred to as total connectedness.

By summarizing column \( i \) (without diagonal elements) one can attain quantification of the time series \( i \)’s total effect on other time series. This is total spillover from time series \( i \) to all other time series. By doing so with row \( i \), it shows the total effect other volatilities have on time series \( i \). This is total spillover to time series \( i \) from all other time series. To achieve net spillover for a time series, one has to net spillover from others and spillover to others. Practically, to
subtract the sum of row $i$ from the sum of column $i$. This value is a measure of how much a time series is a leading or a following player in the analysed set of time series: a positive net value shows net transmitting of volatility, while a negative shows absorption of it.

In my thesis, all values are normalized so that any number in the table tells what part of the forecast error variance of the underlying VAR model concerning a volatility time series is attributable to shocks in another time series. Or to put it another way, what part of the unforecastable volatility of a time series is attributable to own shocks, and what part to shocks in different time series. Tables will be presented in Section 4.
3. Data

In this section, the data used for the analysis is presented. In my paper, I aimed to represent the whole world’s financial market by choosing the right stock exchanges. The chosen stock exchanges need to be scattered all around the globe in a geographical sense, and need to be significant to trustworthily represent the area. Therefore, I chose the biggest stock exchanges in terms of market capitalization in four continents: North-America, South-America, Europe and Asia. Africa and Australia were left out of the analysis as they do not have more than one globally significant stock exchange. And as it had been mentioned in Section 1, a stock index represents a financial market adequately.

So, the data of this paper consists of volatility time series of 16 stock market indices. To obtain volatility time series a range-based volatility estimator is used, as it was discussed in Section 2.2. For choosing the stock indices, I intensively relied on Desjardins’s (2016) work about the world’s most important stock exchanges. Table 1 shows the indices that were chosen, with a short description.

<table>
<thead>
<tr>
<th>STOCK INDEX</th>
<th>Short description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP500</td>
<td>Top 500 stocks traded in NYSE and NASDAQ. (us.spindices.com, 2018)</td>
</tr>
<tr>
<td>NASDAQ</td>
<td>Top 100 stocks traded in NASDAQ stock exchange. (CNBC.com, 2018)</td>
</tr>
<tr>
<td>DOW</td>
<td>Top 30 US Industrial Companies’ stocks. (CNN.com, 2018)</td>
</tr>
<tr>
<td>TSX</td>
<td>Top 250 companies traded in Toronto Stock Exchange. (Toronto Stock Exchange, 2018)</td>
</tr>
<tr>
<td>IBOV</td>
<td>Top 60 stocks traded in B3 stock exchange (Sao Paolo). (bmfbovespa.com, 2018)</td>
</tr>
<tr>
<td>Merval</td>
<td>Top 28 stocks traded in the Buenos Aires Stock Exchange. (topforeignstocks.com, 2018b)</td>
</tr>
<tr>
<td>IPSA</td>
<td>Top 40 stocks traded in the Santiago Stock Exchange. (topforeignstocks.com, 2018b)</td>
</tr>
<tr>
<td>MEXBOL</td>
<td>Top 37 stocks traded in the Mexican Stock Exchange, Mexbol. (BolsaMexicana, 2016)</td>
</tr>
<tr>
<td>FTSE100</td>
<td>Top 100 stocks traded in LSE. (London Stock Exchange, 2018)</td>
</tr>
<tr>
<td>DAX</td>
<td>Top 30 German companies traded in Frankfurt Stock Exchange. (bloomberg.com, 2018)</td>
</tr>
<tr>
<td>CAC40</td>
<td>Top 40 stocks traded in Euronext Paris. (euronext.com, 2018)</td>
</tr>
<tr>
<td>WIG</td>
<td>318 companies traded in the Warsaw Stock Exchange. (investing.com, 2018)</td>
</tr>
<tr>
<td>SHCOMP</td>
<td>Top 1000 companies traded in the Shanghai Stock Exchange. (topforeignstocks.com, 2018a)</td>
</tr>
<tr>
<td>KOSPI</td>
<td>All common stocks traded in the Korea Exchange. (topforeignstocks.com, 2018d)</td>
</tr>
<tr>
<td>HSI</td>
<td>Top 50 stocks traded in the Hong Kong Stock Exchange. (yahoo.com, 2018)</td>
</tr>
<tr>
<td>NIKKEI225</td>
<td>Top 225 companies traded in Tokyo Stock Exchange. (indexes.nikkei.co.jp, 2018)</td>
</tr>
</tbody>
</table>

Table 1: Stock indices in my thesis. 
Sources: Listed separately

For the North American continent, the S&P500 index, the NASDAQ Composite Index, the Dow Jones Industrial Average and the TSX index from Canada have been chosen. In the first 3 stock market index, there are stocks included in 2 of them, which can be a problem. This obviously causes an upward bias in the connectedness of these stock indices, for which one has
to account for accordingly. Apart from that, this causes no problem, and the importance of these stock indices made me use them as instruments of the North-American continent.

Mexico, by the MEXBOL index of the stock exchange of Mexico City is assumed to be in the South-American factor because of cultural ties to Latin-America. A different approach would categorize Mexico to North-America mainly because of geographical reasons, and heavy trade with the US, but I chose the earlier. The stock exchange in Sao Paolo is the most important stock exchange in South-America, it’s index is denoted by IBOV. Argentina’s Buenos Aires Stock Exchange’s index is referred to as MERVAL. Lastly, IPSA is the index of the Santiago Stock Exchange in Chile.

Concerning Europe, FTSE100 represents the stock exchange of London, DAX shows the Frankfurt Stock Exchange, CAC40 the Paris Stock Exchange and WIG the Warsaw Stock Exchange. The first three represent the 3 largest economy in Europe, while WIG is representing a strong Eastern-European economy. By this, not only Western-Europe is represented in the Europe factor, but an effort is made to represent the whole continent. Euronext is neglected because it is not a traditional stock index, and contains stocks from many countries (stoxx.com, 2018).

Talking about Asia, a very unique stock exchange is located in Tokyo. Unique in terms of tending to behave differently than the financial instruments from the rest of the world (Bae & Kim, 2011). The stock index representing Japan is the Nikkei225 index. Apart from Japan, the Chinese financial market possesses a globally important role in Asia. To represent this role, the Shanghai Stock Exchange, and its Composite Index were chosen. SHCOMP is unique in a sense that it is very loosely connected to other stock indices, which will be further explored in my thesis. The HSI index of the Hong Kong Stock Exchange represents another globally important market. The last index from Asia is the KOSPI index of the Seoul Stock Exchange from South-Korea.

For the above indices, daily open, close, high and low prices were downloaded from the Bloomberg terminal at Corvinus University of Budapest. The time-span of the analysis starts at April 2004 and ends at 2018 March. It is important to note however, that if data was missing for a day for any volatility that day must have been excluded from the analysis. This solution is not perfect, but it is even better than using any interpolation method nonetheless. As a consequence, the first day of my research is June 8th, 2004, and the last is March 26th, 2018. Between the two dates, 2563 days had observations for all volatilities.
3.1. Descriptive statistics

For testing normality, a Kolgomorov-Smirnov test (Massey, 1951) was done for each of the volatilities. The test rejects normality on a 5% significance level for every time series. Table 2 shows some basic descriptive statistics about volatilities.

<table>
<thead>
<tr>
<th>INDICES</th>
<th>Mean</th>
<th>Standard dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP500</td>
<td>0.70%</td>
<td>0.58%</td>
<td>0.09%</td>
<td>7.69%</td>
</tr>
<tr>
<td>NASDAQ</td>
<td>0.78%</td>
<td>0.57%</td>
<td>0.13%</td>
<td>6.76%</td>
</tr>
<tr>
<td>DOW</td>
<td>0.69%</td>
<td>0.56%</td>
<td>0.09%</td>
<td>8.55%</td>
</tr>
<tr>
<td>TSX</td>
<td>0.71%</td>
<td>0.62%</td>
<td>0.00%</td>
<td>10.84%</td>
</tr>
<tr>
<td>IBOV</td>
<td>1.24%</td>
<td>0.70%</td>
<td>0.20%</td>
<td>11.37%</td>
</tr>
<tr>
<td>MERVAL</td>
<td>1.18%</td>
<td>0.74%</td>
<td>0.19%</td>
<td>12.14%</td>
</tr>
<tr>
<td>IPSA</td>
<td>0.62%</td>
<td>0.42%</td>
<td>0.09%</td>
<td>5.46%</td>
</tr>
<tr>
<td>MEXBOL</td>
<td>0.83%</td>
<td>0.52%</td>
<td>0.18%</td>
<td>6.55%</td>
</tr>
<tr>
<td>FTSE100</td>
<td>0.77%</td>
<td>0.54%</td>
<td>0.15%</td>
<td>6.10%</td>
</tr>
<tr>
<td>DAX</td>
<td>0.92%</td>
<td>0.62%</td>
<td>0.10%</td>
<td>6.42%</td>
</tr>
<tr>
<td>CAC40</td>
<td>0.89%</td>
<td>0.59%</td>
<td>0.15%</td>
<td>6.45%</td>
</tr>
<tr>
<td>WIG</td>
<td>0.73%</td>
<td>0.48%</td>
<td>0.13%</td>
<td>5.52%</td>
</tr>
<tr>
<td>SHCOMP</td>
<td>1.13%</td>
<td>0.74%</td>
<td>0.16%</td>
<td>6.34%</td>
</tr>
<tr>
<td>KOSPI</td>
<td>0.77%</td>
<td>0.57%</td>
<td>0.14%</td>
<td>10.10%</td>
</tr>
<tr>
<td>HSI</td>
<td>0.78%</td>
<td>0.54%</td>
<td>0.18%</td>
<td>10.18%</td>
</tr>
<tr>
<td>NIKKEI225</td>
<td>0.75%</td>
<td>0.52%</td>
<td>0.11%</td>
<td>7.84%</td>
</tr>
</tbody>
</table>

Table 2: Basic descriptive statistics of daily volatilities.
Source: Bloomberg, own calculations

Figure 1: S&P500, IBOV, FTSE100 and SHCOMP volatilities, June 2004 - March 2018
Source: Bloomberg (2018), own calculations
On Figure 1 plot of 4 volatilities (1 from each continent) are shown for the full time-span. Only these four are shown because these are enough to demonstrate my point. It is important to note the crisis around 2008, which produces a spike in all the time series in the middle of the time-span. Concerning IBOV, the spike during the crisis is much larger than spikes in other volatility time series. It seems, that the 2007-2009 crisis caused a larger volatility increase in the Brazilian financial markets than in other markets. Another spike worth noting is in FTSE100 around the end of the time-span which is caused by Brexit (BBC.com, s.a.). Looking at SHCOMP it is striking how low its volatility’s persistence is, to how much of an extent its volatility is volatile.

For all the above reasons, and for the purposes of better estimating the models, the volatilities are standardized. Figure 2 shows S&P500 volatility histograms before and after standardizing the time series. The distribution of volatilities is positively skewed, with a value being 4.18. This means that days with extremely high volatility are far more frequent than days with low volatility. This can be seen on Figure 2 as well.

![Figure 2: Histogram of S&P500 volatilities before and after standardization. (outliers left out for visibility)
Source: Bloomberg, own calculations](image)
4. Estimation and results

In this section the results of the analysis are presented. Firstly, my estimation strategy is presented. Secondly, I will show how the global-local model drawn in Section 2 performed in extracting the global factor, and compare it to the global factor attained by PCA. Then shortly discuss the equations of the Global-Local model, and continue with the Global-Regional-Local model. After that, variance decompositions are made in order to quantify the importance of global, regional and local factors in the volatility of each stock market index. After that, the Diebold-Yilmaz Spillover Framework is applied to the time series of volatility, and to the local factors – from which the effect of global and regional causes is extracted. Subsection 4.7 concludes with a dynamic analysis of total connectedness and regional net spillover.

4.1. Estimation strategy

First, the estimation strategy of the Global-Local model is presented, while at the end of the present subsection, important differences in estimating the Global-Regional-Local model is pointed out. “The estimation of the Global Factors is made by Kalman Filter. However, the methodology is very sensitive to the initial values. Therefore, there are a series of intermediate steps before the final estimation.” (Morita & Bueno, 2008, p. 10). Therefore, some preparation has to be done, before estimating the state-space model. My econometric strategy, as Morita & Bueno (2008) calls it is very similar to theirs. First of all, a good first guess for the global volatility factor would be the first component of result of the Principal Component Analysis (from now on: PCA).

PCA’s purpose is “from variables that are linearly correlating with each other pairwise through orthogonal transformation we can make principal components so that the first some component explains a sufficiently large portion of the variance of the variables, thereby transforming our observations to less dimensions” (Kovács, 2014, p. 149, own translation). An important premise of efficiently using PCA is that the observations should correlate with each other. Correlations between volatilities are shown in Table 4. The correlations are between 0.15 and 0.98, with a mean of 0.54, which is encouraging.
Table 3: Descriptive statistics of stock index correlations.
Source: Bloomberg (2018), own calculations

<table>
<thead>
<tr>
<th>CORRELATION</th>
<th>FULL</th>
<th>EXCL. SHCOMP</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum</td>
<td>0.15</td>
<td>0.33</td>
</tr>
<tr>
<td>maximum</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>mean</td>
<td>0.54</td>
<td>0.58</td>
</tr>
<tr>
<td>median</td>
<td>0.54</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Table 4 shows the correlation matrix, while Table 3 shows descriptive statistics of correlations between stock indices on the full time-span. It can be seen, that there are significant correlations, and if SHCOMP index, which has very weak ties to the other indices is excluded, the minimum correlation increases from 0.15 to 0.33. This also shows signs of the existence of relevant common factors.

The first principal component includes 59.03% of the information from the 16 volatility time series, which is also a good sign concerning the global volatility factor. Subsequently, an autoregressive model was fit on the time series of the first principal component. It is a first order AR, so it uses one lag to explain the forthcoming element. The equation of the model can be written as Equation 13.

\[
P_{\text{CA} \text{Global}}_t = \varphi \times P_{\text{CA} \text{Global}}_{t-1} + \tau_t,
\]

where \(\varphi\) is the autoregressive coefficient which will be the initial value of the first element of the diagonal of matrix A. \(\tau_t\) is a regular white noise error term.
<table>
<thead>
<tr>
<th></th>
<th>SP500</th>
<th>NASDAQ</th>
<th>DOW</th>
<th>TSX</th>
<th>IBOV</th>
<th>MERVAL</th>
<th>IPSA</th>
<th>MEXBOL</th>
<th>FTSE100</th>
<th>DAX</th>
<th>CAC40</th>
<th>WIG</th>
<th>SHCOMP</th>
<th>KOSPI</th>
<th>HSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>DOW</td>
<td>0.98</td>
<td>0.91</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
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<tr>
<td>IBOV</td>
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<tr>
<td>MERVAL</td>
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<td></td>
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</tr>
<tr>
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<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>MEXBOL</td>
<td>0.73</td>
<td>0.71</td>
<td>0.73</td>
<td>0.70</td>
<td>0.67</td>
<td>0.47</td>
<td>0.56</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>FTSE100</td>
<td>0.74</td>
<td>0.71</td>
<td>0.74</td>
<td>0.70</td>
<td>0.58</td>
<td>0.42</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DAX</td>
<td>0.73</td>
<td>0.71</td>
<td>0.73</td>
<td>0.67</td>
<td>0.54</td>
<td>0.41</td>
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<tr>
<td>CAC40</td>
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</tr>
<tr>
<td>WIG</td>
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<td>0.58</td>
<td>0.58</td>
<td>0.57</td>
<td>0.50</td>
<td>0.34</td>
<td>0.46</td>
<td>0.56</td>
<td>0.63</td>
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<td>0.60</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>SHCOMP</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.21</td>
<td>0.15</td>
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<td>0.29</td>
<td>0.26</td>
<td>0.25</td>
<td>0.22</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>KOSPI</td>
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<td>0.58</td>
<td>0.58</td>
<td>0.54</td>
<td>0.46</td>
<td>0.34</td>
<td>0.40</td>
<td>0.54</td>
<td>0.58</td>
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</tr>
<tr>
<td>HSI</td>
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<td>0.54</td>
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<td></td>
</tr>
<tr>
<td>NIKKEI225</td>
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<td>0.49</td>
<td>0.43</td>
<td>0.35</td>
<td>0.40</td>
<td>0.49</td>
<td>0.53</td>
<td>0.49</td>
<td>0.46</td>
<td>0.38</td>
<td>0.26</td>
<td>0.54</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Table 4: Correlations between stock indices.
Source: Bloomberg, own calculations
The additional 16 initial values for matrix A’s diagonal are obtained using a vector autoregressive (VAR) model (Enders, 2010). VAR models are used for forecasting the vector of time series from lagged values of the same vector of time series. Therefore, this VAR model produces coefficients about the time series affecting each other (16x16 matrix). But as I assume, that non-diagonal elements of the coefficient matrix show connections through the global or regional volatility factor, only the diagonal elements of this coefficient matrix are important for me. In addition, if all the elements of matrix A, B, C, D were to be estimated would increase the number of estimated parameters drastically. These 16 diagonal coefficients will be the additional 16 initial values for matrix A’s diagonal. The other values of matrix A was set to 0.

About matrix B, initial values are obtained from the above-mentioned AR and VAR model. The diagonal of the covariance matrix of error terms in these models are used for initial values in matrix B’s diagonal.\(^3\) The other values of matrix B was set to 0.

Concerning matrix C and D, additional regression analysis needed to be done. The volatility time series are regressed against the first principal component, our best guess of the global volatility factor (by now). The regression equation is shown in Equation 14.

\[
\text{vol}_{i,t} = \alpha_i + \beta_i \times \text{PCA}_{Global,t} + \delta_t
\]

In Equation 14 \(\text{vol}_{i,t}\) is the value of volatility time series \(i (i=1,2\ldots16)\) in time \(t\), the other parts are known by now. These 16 equations produce 16 betas which will be initial values for matrix C’s first column (\(\beta_i\) for C(i, 1)). The first column of C shows how the global factor affects the observed volatilities, hence the initial values’ obtaining method. The diagonal of the remaining of matrix C is set to 1, and other elements to 0. Thereby only the corresponding states affect the measured volatilities, and of course, the global factor.

For matrix D, obtaining the initial values are very similar to the case of matrix B. The square root of the diagonal elements of the covariance matrix of the innovation matrix of Equation 14 are used for initial values in matrix D’s diagonal. The calculations were made on Matlab2017a software.

In the case of the Global-Regional-Local model, the initial guesses for the regional factors are obtained by regressions, too. In this case, a PCA is conducted on the 16 volatilities, and one PCA for each of the 4 regional subset. Subsequently, the first component of the regional PCAs

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\(^3\) To be precise, the square roots of these values are used.
are regressed against the first component of the PCA performed on all the volatilities. The residuals of these regressions are initial guesses for the regional factors. By this method, one is extracting the global information, ending up with only the regional common factors.

Fitting an AR model on these initial regional factors produces additional initial values in matrix A’s diagonal – just like it did in the case of the global factor in the Global-Local model. Equation 14 in the Global-Regional-Local model can be formulated as Equation 15. \( \beta_{i,G} \) are initial values for the first column of matrix C, and \( \beta_{i,R} \) are initial values for the non 0 elements of columns 2-5. Other aspects of the estimation strategy is similar to the Global-Local model.

\[
\text{vol}_{i,t} = \alpha_i + \beta_{i,G} \cdot \text{PCA}_{Global,t} + \beta_{i,R} \cdot \text{PCA}_{Regional,t} + \delta_t
\]

4.2. State-space results – the Global-Local model

The first result to present is the fitted Global-Local model. On Figure 3, the global factors filtered out from the volatility of 16 stock market indices are shown. It is evident from the picture, that the PCA-based global factor was definitely close to the more sophisticated state space approach’s global factor. The correlation between the two is around 99%, with the state-space model producing a somewhat smoother factor. A smoother factor is more favourable as it shows that it is less moved by sudden shocks, and rather by important underlying processes in volatilities – a desirable attribute for a latent factor.

Some days with considerable spikes are worth noting. The highest spike in Figure 3 is the period of the Great Recession (Rich, 2013) of late 2008-2009. Two extremely high values are 10th and 24th of October 2008. The 10th of October is a day of unprecedented events since 1930: on that day, Henry Paulson, Secretary of the Treasury in the Bush administration, former investment banker at Goldman Sachs announced the purchase of financial firms’ securities by the government (The Associated Press, 2008). On top of that, 24th of October can be considered peak of the crisis in some sense, with the world’s leaders consulting in Beijing the next day about necessary measures treating the global crisis (Bradsher, 2008). Another spike is observable at 24th June 2016 is the day following the referendum resulting in the UK leaving the European Union (BBC.com, s.a.).

On these days, the standardized value of the global volatility factor (the PCA and state-space model based as well) has a higher value than any of the volatilities. This suggests that the global factor’s deviation from its mean in standard deviation units is higher than that of single
volatilities. That fact suggests the role of the global factor appreciates in times of crises. This definitely means higher connectedness between financial markets, and it means higher connectedness through the global factor.

![Graph showing global factors, Global-Local model, 2004-2018](image)

*Figure 3: Global factors using PCA and state-space model in 2004-2018. Source: Bloomberg, own calculations*

The state-space systems equations are shown in Figure 4. Concerning the global factor, it is persistent which is shown by its high coefficient. The importance of the global factor concerning stock index volatilities are shown in the respective betas for the global factor.

In terms of betas for the global factor, the analysed stock indices show variation between 0.12 and 0.3. The lowest is SHCOMP, which is an early sign, that the Chinese financial markets behave substantially different. The importance of common factors can also be seen in the persistence of $\epsilon_t$. In this sense, SHCOMP excel again with fellow Asian stock indices. An encouraging sign for a Global-Regional-Local model is that stock indices in the same continent have persistence closer to each other than to volatilities in other continents in general. North-America’s values are between 0.38 and 0.51, while in Asia these values are in the 0.72-0.92 range.
Global-Local model

\[ G_t = 0.83 \cdot G_{t-1} + \nu_t \]

S&P500_t = 0.30 \cdot G_t + \epsilon_{S&P500_t} \\
NASDAQ_t = 0.29 \cdot G_t + \epsilon_{NASDAQ_t} \\
DOW_t = 0.30 \cdot G_t + \epsilon_{DOW_t} \\
TSX_t = 0.28 \cdot G_t + \epsilon_{TSX_t} \\
IBOV_t = 0.24 \cdot G_t + \epsilon_{IBOV_t} \\
MERVAL_t = 0.19 \cdot G_t + \epsilon_{MERVAL_t} \\
IPSA_t = 0.21 \cdot G_t + \epsilon_{IPSA_t} \\
IXMEXt = 0.26 \cdot G_t + \epsilon_{IXMEXt} \\
FTSE100_t = 0.28 \cdot G_t + \epsilon_{FTSE100_t} \\
DAX_t = 0.29 \cdot G_t + \epsilon_{DAX_t} \\
CAC40_t = 0.27 \cdot G_t + \epsilon_{CAC40_t} \\
WIG_t = 0.23 \cdot G_t + \epsilon_{WIG_t} \\
SHCOMP_t = 0.12 \cdot G_t + \epsilon_{SHCOMP_t} \\
KOSPI_t = 0.23 \cdot G_t + \epsilon_{KOSPI_t} \\
HSI_t = 0.22 \cdot G_t + \epsilon_{HSI_t} \\
NIKKE125t = 0.21 \cdot G_t + \epsilon_{NIKKE125t}

\[ \epsilon_{S&P500_t} = 0.38 \cdot \epsilon_{S&P500_{t-1}} + \eta_{S&P500_t} \] \\
\[ \epsilon_{NASDAQ_t} = 0.45 \cdot \epsilon_{NASDAQ_{t-1}} + \eta_{NASDAQ_t} \] \\
\[ \epsilon_{DOW_t} = 0.40 \cdot \epsilon_{DOW_{t-1}} + \eta_{DOW_t} \] \\
\[ \epsilon_{TSX_t} = 0.51 \cdot \epsilon_{TSX_{t-1}} + \eta_{TSX_t} \] \\
\[ \epsilon_{IBOV_t} = 0.65 \cdot \epsilon_{IBOV_{t-1}} + \eta_{IBOV_t} \] \\
\[ \epsilon_{MERVAL_t} = 0.80 \cdot \epsilon_{MERVAL_{t-1}} + \eta_{MERVAL_t} \] \\
\[ \epsilon_{IPSA_t} = 0.75 \cdot \epsilon_{IPSA_{t-1}} + \eta_{IPSA_t} \] \\
\[ \epsilon_{IXMEXt} = 0.60 \cdot \epsilon_{IXMEX_{t-1}} + \eta_{IXMEXt} \] \\
\[ \epsilon_{FTSE100_t} = 0.50 \cdot \epsilon_{FTSE100_{t-1}} + \eta_{FTSE100_t} \] \\
\[ \epsilon_{DAX_t} = 0.53 \cdot \epsilon_{DAX_{t-1}} + \eta_{DAX_t} \] \\
\[ \epsilon_{CAC40_t} = 0.55 \cdot \epsilon_{CAC40_{t-1}} + \eta_{CAC40_t} \] \\
\[ \epsilon_{WIG_t} = 0.70 \cdot \epsilon_{WIG_{t-1}} + \eta_{WIG_t} \] \\
\[ \epsilon_{SHCOMP_t} = 0.92 \cdot \epsilon_{SHCOMP_{t-1}} + \eta_{SHCOMP_t} \] \\
\[ \epsilon_{KOSPI_t} = 0.71 \cdot \epsilon_{KOSPI_{t-1}} + \eta_{KOSPI_t} \] \\
\[ \epsilon_{HSI_t} = 0.72 \cdot \epsilon_{HSI_{t-1}} + \eta_{HSI_t} \] \\
\[ \epsilon_{NIKKE125t} = 0.77 \cdot \epsilon_{NIKKE125_{t-1}} + \eta_{NIKKE125t} \]

Figure 4: State-space equations in the Global-Local model. Source: Bloomberg, own calculations

After estimating the global factor from volatilities, one can filter the global factor from these time series. This is done by the regression shown by Equation 6. Concerning local factors, one would expect them to be correlated weakly, or even negatively, because the common factors are extracted in the global factor. Table 4 shows the average correlation of local factors within and outside a region. This is evident, that there are some regional communalities that our model failed to account for. In the Global-Regional-Local model, this analysis is continued.

\[ \text{volatility}_{i,t} = \alpha_i + \beta_i \cdot \text{SSMGlobal}_t + \text{LocalFactor}_{i,t} \]
4.3. State-space results – the Global-Regional-Local model

In the following Subsection the results of the state-space model in Global-Regional-Local settings are presented. Figure 5 shows the global factor created with the two different methodologies like it was presented in the previous Subsection. The correlation between the global factor (of the state-space model) in this setting and the global factor extracted in the Global-Local setting is 0.99. Therefore, the previous setting estimated the global factor well enough, and information in the regional factors are mostly extracted from the local factors.

<table>
<thead>
<tr>
<th>REGION</th>
<th>Average correlation</th>
<th>Within region</th>
<th>Outside region</th>
</tr>
</thead>
<tbody>
<tr>
<td>North-America</td>
<td>0.37</td>
<td>-0.13</td>
<td></td>
</tr>
<tr>
<td>South-America</td>
<td>0.14</td>
<td>-0.06</td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td>0.31</td>
<td>-0.15</td>
<td></td>
</tr>
<tr>
<td>Asia</td>
<td>0.19</td>
<td>-0.09</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Correlations between local factors of stock indices, by region.
Source: Bloomberg (2018), own calculations

In Figures 6-9 the extracted regional factors are shown. All the regional factors have significantly lower variation than the global factor. It also seems clear, that during quiet times...
the regional factors are very close to zero, while during crises their value goes up – in this sense, the regional factors are very similar to each other. On the other hand, all regional factors – particularly the European and the Asian factor has sudden spikes, which are hard to interpret.

One possible explanation for the nature of regional factors is as follows. During quiet times, because of the extent globalization is present in the world, stock markets behave quite a similar way – at least in terms of stock market volatility, and at least regional stock exchanges do not have more in common, than with any global stock exchange. However, during crisis – while the global factor’s value appreciates as well – some regional communalities emerge. Global crises can hit different geographical regions differently because of social, macroeconomic or other reasons, and that is what is reflected in the regional factors’ dynamics.

![North-America factors, Global-Regional-Local model, 2004-2018](image)

*Figure 6: North-America factors using PCA and state-space model in 2004-2018.*
*Source: Bloomberg, own calculations*
Figure 7: North-America factors using PCA and state-space model in 2004-2018. Source: Bloomberg, own calculations

Figure 8: North-America factors using PCA and state-space model in 2004-2018. Source: Bloomberg, own calculations
On Table 6., the correlation between global and regional factors extracted using state-space models and PCA are shown. The global factor’s correlation is somewhat lower than it was in the Global-Local model setting. Regarding regional factors, the picture is more mixed. The low correlation of the North-American factor tells us, that the more sophisticated methodology of state-space models in that case had a high value added. This is true however to a lesser extent to Europe and South-America as well, and somewhat to Asia, too.

<table>
<thead>
<tr>
<th>FACTORS</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global</td>
<td>96.99%</td>
</tr>
<tr>
<td>North-America</td>
<td>72.53%</td>
</tr>
<tr>
<td>South-America</td>
<td>84.24%</td>
</tr>
<tr>
<td>Europe</td>
<td>84.33%</td>
</tr>
<tr>
<td>Asia</td>
<td>92.30%</td>
</tr>
</tbody>
</table>

*Table 6: Correlations between factors produced by PCA and by SSM. (2004-2018)*

*Source: Bloomberg, own calculations*
Looking at the equations driving the Global-Regional-Local model the previously seen attributes can be seen again. The global factor has a relatively high persistence, while the regional factors’ autoregressive coefficients are between 0.2 and 0.38, North-America having the lowest while South-America having the highest.

Volatility betas for global and regional factors are close to each other. IPSA and SHCOMP having the lowest beta for the global factor, while S&P500 and FTSE100 having the highest beta. Regional factor’s higher beta does not mean a more important role – as we will see in Subsection 4.4.
one among the highest for the regional factor. This is not true for SHCOMP, nonetheless it has a middle tier coefficient.

<table>
<thead>
<tr>
<th>INDICES</th>
<th>Global-Local</th>
<th>Global-Regional-Local</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP500</td>
<td>0.38</td>
<td>0.07</td>
<td>-0.32</td>
</tr>
<tr>
<td>NASDAQ</td>
<td>0.45</td>
<td>0.26</td>
<td>-0.19</td>
</tr>
<tr>
<td>DOW</td>
<td>0.40</td>
<td>0.13</td>
<td>-0.27</td>
</tr>
<tr>
<td>TSX</td>
<td>0.51</td>
<td>0.22</td>
<td>-0.29</td>
</tr>
<tr>
<td>IBOV</td>
<td>0.65</td>
<td>0.54</td>
<td>-0.11</td>
</tr>
<tr>
<td>MERVAL</td>
<td>0.81</td>
<td>0.33</td>
<td>-0.47</td>
</tr>
<tr>
<td>IPSA</td>
<td>0.75</td>
<td>0.42</td>
<td>-0.33</td>
</tr>
<tr>
<td>MEXBOL</td>
<td>0.59</td>
<td>0.38</td>
<td>-0.21</td>
</tr>
<tr>
<td>FTSE100</td>
<td>0.50</td>
<td>0.39</td>
<td>-0.11</td>
</tr>
<tr>
<td>DAX</td>
<td>0.53</td>
<td>0.20</td>
<td>-0.33</td>
</tr>
<tr>
<td>CAC40</td>
<td>0.56</td>
<td>0.29</td>
<td>-0.26</td>
</tr>
<tr>
<td>WIG</td>
<td>0.70</td>
<td>0.23</td>
<td>-0.47</td>
</tr>
<tr>
<td>SHCOMP</td>
<td>0.92</td>
<td>0.79</td>
<td>-0.14</td>
</tr>
<tr>
<td>KOSPI</td>
<td>0.71</td>
<td>0.54</td>
<td>-0.17</td>
</tr>
<tr>
<td>HSI</td>
<td>0.72</td>
<td>0.35</td>
<td>-0.37</td>
</tr>
<tr>
<td>NIKKEI225</td>
<td>0.77</td>
<td>0.48</td>
<td>-0.29</td>
</tr>
</tbody>
</table>

Table 7: Autoregressive coefficients of local factors in model settings.
Source: Bloomberg, own calculations

Another important aspect is the persistence of local factors. This can be measured by the autoregressive coefficients on Figure 10’s right-hand side. Here, SHCOMP – as it was expected – excel, while SP500 has the lowest persistence. The explanation for the extremely low persistence of the local factor of SP500 can be its global leading role – very few information affecting this index can be considered local.

Table 7 shows autoregressive coefficients of local factors in the two different model settings. Additional information captured in regional factors can be seen by the fact that coefficients in the Global-Regional-Local model are lower in the case of every stock index.

4.4. Variance decomposition

In the previous Subsection it was highlighted that autoregressive coefficients do not tell much about how much a volatility is driven by the global or the corresponding regional factor. However, variance decomposition is exactly a tool for this. In this Subsection I will show what parts of each volatility time series variance are attributable to global, local and idiosyncratic factors.
Variance decomposition is done by comparing variance of time series. To the total variance of volatility time series, the variance of residuals is compared. The global factor’s role is obtained through comparing the variance of residuals of regression formulated as in Equation 14 to the total variance of volatility time series. The part of variance explained by the global factor, is the variance part missing from the residuals.

The regional factors’ share can be obtained by running regression with the global and regional factors, as in Equation 15. The regional factor’s share is the additional explained variance compared to the previous regression setting. Results are presented in Tables 8-11, separated for each continent.

<table>
<thead>
<tr>
<th>VARIANCE SHARES</th>
<th>SP500</th>
<th>NASDAQ</th>
<th>DOW</th>
<th>TSX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global factor</td>
<td>80.69%</td>
<td>76.87%</td>
<td>77.12%</td>
<td>73.73%</td>
</tr>
<tr>
<td>North American factor</td>
<td>18.87%</td>
<td>11.07%</td>
<td>21.20%</td>
<td>1.94%</td>
</tr>
<tr>
<td>Local factor</td>
<td>0.44%</td>
<td>12.07%</td>
<td>1.68%</td>
<td>24.33%</td>
</tr>
</tbody>
</table>

*Table 8: Variance decomposition of North-American stock indices. Source: Bloomberg (2018), own calculations*

<table>
<thead>
<tr>
<th>VARIANCE SHARES</th>
<th>IBOV</th>
<th>Merval</th>
<th>IPSA</th>
<th>MEXBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global factor</td>
<td>51.12%</td>
<td>27.73%</td>
<td>35.71%</td>
<td>58.93%</td>
</tr>
<tr>
<td>South American factor</td>
<td>30.68%</td>
<td>24.44%</td>
<td>21.65%</td>
<td>7.96%</td>
</tr>
<tr>
<td>Local factor</td>
<td>18.20%</td>
<td>47.83%</td>
<td>42.63%</td>
<td>33.11%</td>
</tr>
</tbody>
</table>

*Table 9: Variance decomposition of South-American stock indices. Source: Bloomberg (2018), own calculations*

Regarding the North-American continent, the global factors role is the most important among all continents. This can be interpreted as a sign, that the global factor is mainly shaped by events of this continent. Taking the USA-s role in the global economy into account, this interpretation sounds plausible. The regional factors role is significantly larger in the U.S. based indices than in TSX – I assume that this is at least partly because the above mentioned same stocks in these indices.

The global factors role in South-American indices shows much more variation. The globally much more relevant IBOV and MEXBOL indices are experiencing much more connection with the global factor, while in the case of Merval and IPSA, the local factor keeps almost half of the variance. Low importance of the regional factor in the case of MEXBOL may point to the fact, that my decision of assuming it to be a South-American stock index was not perfect.
Table 10: Variance decomposition of European stock indices. 
Source: Bloomberg (2018), own calculations

Table 11: Variance decomposition of Asian stock indices. 
Source: Bloomberg (2018), own calculations

Concerning Europe, one can see the difference between the Western-European stock indices and the Eastern-European WIG index. FTSE100, DAX and CAC40 are quite similar looking at the global factors role, while the regional factor is stronger in the case of indices of Continental Europe. The stronger local factor in the case of the FTSE100 can be a sign, that this stock index is just as connected to North-American indices because of socio-cultural reasons. This can be seen by simple correlations as well.

The variance decomposition Asian stock indices reflect the different economic conditions these countries experience. The stock indices of relatively open, western oriented countries of South-Korea and Hong Kong are more tied to the global factor. The regional factor is strongest in the case of HSI – showing a regional leading role. However, it is the least relevant for NIKKEI225, where the global factor’s role is above expectations. If one is looking for a stock index that is relatively independent from global factors, the SHCOMP index emerges as a better solution.

4.5. Spillover between volatilities

In the following two Subsections, the Diebold-Yilmaz framework is applied to volatility time series, and later on local factor time series from the Global-Regional-Local model setting. In this case, the Diebold-Yilmaz spillover framework was applied to the time series of volatilities. The predicting horizon was 10 days throughout this paper, as in most of the time the framework is applied, for example in Diebold & Yilmaz (2009) or Demirer et al. (2017). This is sensible because of the fact that stock exchanges being closed on the weekend makes 10 days exactly two weeks’ time. The VAR lag was chosen to be 2, mainly because – as robustness checks will
show – using a first order VAR causes substantially different results than using any other, and because 2 is used by previous analyses (Diebold & Yilmaz, 2009).

Table 14 shows the spillover table of volatilities for the total time-span. The total connectedness of the time series is 0.74 which is relatively high compared to Diebold & Yilmaz’s (2009) results, who also analysed stock market indices. Their total connectedness was 0.395 in terms of volatility. Concerning the net connectedness, North-American indices are being massive net transmitters of volatility, which is in accordance with previous findings. FTSE100 and MEXBOL are the only non-North-American index net transmitting volatility. Asian indices are substantial receivers of volatility, while South-American indices are quite even except IPSA, which is also a receiver. European indices apart from FTSE100 are among the relatively even receivers and transmitters, WIG only receiving a significant amount of its volatility from others.

SHCOMP index has extremely low spillover values, relatively uniquely. This may be because of the Chinese markets having different characteristics and moving quite separately from the other parts of the world, a fact that we previously saw looking at volatility correlations, and state-space models as well. Also, the state-driven nature of the Chinese economy, along with the industrial components of SHCOMP provides explanation for the outstanding nature of this index.

Nikkei225 is only different because it is receiving more volatility. Japan being different from the others is also an unsurprising finding, Diebold et al. (2008) showed that in terms of yield curve dynamics, and Diebold & Yilmaz (2009) concerning stock indices as well. Apart from SHCOMP, the other indices are relatively close to each other in receiving volatility (0.6-0.83), while in transmitting they show much more variation. On transmitting, North-American indices and FTSE100 excel from the others.
Figure 11 visualizes Table 14 as a network. The visualization will happen along the same characteristics throughout this thesis. Arrows indicate directions of volatility spillover. Edge sizes and colours are based on spillover: the darker (red) and thick an edge is, the stronger spillover in that direction is. Node size is based on the average volume the stock index is traded in a day (Bloomberg, 2018). Node colour is based on net spillover: negative net spillover (receiving volatility) means purple, positive (transmitting volatility) means green. The arrangement of the network was done by the Force Atlas 2 algorithm. The layout of the graph also shows edge strength: the stronger an edge is (stronger spillover is), the closer the nodes are. For visualizing the network, Gephi 0.9.2 software was used. Connections lower than 0.1 are filtered out from the graph.

The uniqueness of the Chinese market is dazzlingly clear from this graph. The Japanese index is relatively alone, with much stronger arrows pointing towards it, than from it to others. It is also no wonder, that North-American indices are to be found at the centre of the network, in
greenish colour, stronger arrows pointing from it to others than the other way around. Regions form clusters, Asia being the least clear. Europe and South-America has a node (FTSE100 and MEXBOL, respectively) that is closer to the centre, than other indices from the continent – making them bridge nodes. In the next subsection, the same analysis is made on the local volatility factors obtained in the previous subsection.

4.6. Spillover between local factors

The time series of local volatility factors are obtained by regressing the volatilities on the global and regional factors. From this regression, the residuals are the parts of the volatility that cannot be explained by the global and regional factors – therefore it must be local in my setting. Equation 11 shows the way local factors are the residuals after filtering out the global factor from volatilities.

Table 15 shows the spillover table for local volatility factors in the full time-span. The total connectedness is 0.23 which shows that the global factor was responsible for roughly two-thirds of the total connectedness. An interesting result is that virtually all local factors transmits roughly as much volatility as they receive. In terms of local factor volatility spillover, the world seems like a more equal place. SHCOMP, NKY and ASX200 is relatively separate from the others again, in terms of receiving and transmitting as well. This may mean that volatility transmitting and receiving is mainly done by shaping the global and regional volatility factors. Figure 12 shows the same network graph for local factors.
The graph in Figure 12 a little different from the one can be seen concerning volatilities. While in Figure 11 15 indices were relatively close to each other, only SHCOMP being totally separated from the others, in Figure 12 regional stock indices are clustered together, with globally important nodes forming between-continent ties.
### Table 12-13: Spillover table for volatilities (upper) and for local factors. Sources: Bloomberg (2018), own calculations.

<table>
<thead>
<tr>
<th>SP500</th>
<th>SP500</th>
<th>NASDAQ</th>
<th>DOW</th>
<th>TSX</th>
<th>IBOV</th>
<th>Merval</th>
<th>IPSA</th>
<th>MEXBOL</th>
<th>FTSE100</th>
<th>DAX</th>
<th>CAC40</th>
<th>WIG</th>
<th>SHCOMP</th>
<th>KOSPI</th>
<th>HSI</th>
<th>NIKKEI225</th>
<th>FROM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.17</td>
<td>0.13</td>
<td>0.16</td>
<td>0.10</td>
<td>0.06</td>
<td>0.04</td>
<td>0.03</td>
<td>0.06</td>
<td>0.07</td>
<td>0.05</td>
<td>0.05</td>
<td>0.03</td>
<td>0.00</td>
<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
<td>0.83</td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>0.17</td>
<td>0.14</td>
<td>0.09</td>
<td>0.05</td>
<td>0.04</td>
<td>0.03</td>
<td>0.06</td>
<td>0.06</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.03</td>
<td>0.01</td>
<td>0.02</td>
<td>0.02</td>
<td>0.83</td>
<td></td>
</tr>
<tr>
<td>0.16</td>
<td>0.12</td>
<td>0.17</td>
<td>0.10</td>
<td>0.06</td>
<td>0.04</td>
<td>0.03</td>
<td>0.06</td>
<td>0.07</td>
<td>0.05</td>
<td>0.05</td>
<td>0.03</td>
<td>0.01</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.83</td>
<td></td>
</tr>
<tr>
<td>0.12</td>
<td>0.09</td>
<td>0.12</td>
<td>0.20</td>
<td>0.06</td>
<td>0.05</td>
<td>0.04</td>
<td>0.06</td>
<td>0.07</td>
<td>0.05</td>
<td>0.05</td>
<td>0.03</td>
<td>0.01</td>
<td>0.03</td>
<td>0.02</td>
<td>0.01</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>0.08</td>
<td>0.10</td>
<td>0.24</td>
<td>0.07</td>
<td>0.04</td>
<td>0.08</td>
<td>0.05</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.02</td>
<td>0.00</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td>0.07</td>
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**Total connectedness: 0.74**

| NET     | 0.59    | 0.33    | 0.57   | 0.38   | -0.01  | -0.05  | -0.22  | 0.08   | 0.12   | -0.07  | -0.31  | -0.24  | -0.27  | -0.35  | -0.43  | 0.74  |

**Total connectedness: 0.74**
4.7. Dynamic analysis

The last Subsection of results of my thesis arises from a dynamic analysis of stock market volatility connectedness. Firstly, total connectedness is analysed for volatilities and for local factors as well. The rolling window is 1 year (220-day approximation because of missing data) long, and could be fit to the data 2344 times. In every period, total connectedness was calculated for volatilities and for local factors. The results are shown on Figure 13. The date marks the start of the period.

![Total connectedness for volatilities and local factors, rolling window](image)

**Figure 13:** Total connectedness between volatilities and local volatility factors, 2004-2018, yearly rolling window

Source: Bloomberg, own calculations

The results are somewhat different from the expected. One would wait the total connectedness of the local factors to be significantly lower than that of volatilities in any time period, because the global factor which is connectedness itself, had been extracted. However, for a short period local component connectedness is even greater than volatility connectedness. But an interesting phenomenon arises if one looks at total connectedness of volatilities and the global factor of the Global-Regional-Local model simultaneously.

Figure 14 shows the difference between the 220-day rolling window estimated total connectedness (like in Figure 13 in blue) of volatilities and local factors, and the 220-day forward looking moving average of the global factor of the Global-Regional-Local model. The correlation of them is 50%, and correlation between total connectedness and the moving
average of the global factor is 57%. It can be seen, that there is a connection between the two, and my results reinforce each other in this case.

Figure 14: Total connectedness of volatilities (left axis) and the 220-day moving average of the global factor.
Source: Bloomberg (2018), own calculations

Secondly, the net spillover of regions is explored. Net spillover of a region can be defined as the sum of directional spillover of regional indices to indices outside the region, minus the same from indices outside the region, and divided by 4 (the number of stock indices in a continent), to achieve comparability (transform to 0-1 range). On Figure 15-16 regional net spillover of volatilities and local factors (respectively) are plotted.

Figure 15: Regional net spillover of volatilities, 220-day rolling window, 2004-2018
Source: Bloomberg (2018), own calculations
In terms of regional net spillover of volatilities the first thing that can be seen is North-America’s net spillover constantly below 0. This means that their high net spillover is mainly because of volatility transmission inside the region – a finding reinforced by the spillover table as well. It is also easy to see from the figures, that some regions have a strong connection with each other. Correlations of regional net spillover are shown in Table 14.

The strong negative correlation between net spillover of Asia and North-America, and between Europe and South-America in terms of spillover is an interesting phenomenon. Regarding local factors, Europe and Asia experience significant negative correlation – however in terms of volatility they are positively correlated. The same change is happening with North-America and South-America. The latter is due to the extraction of the global factor, which hid the negative correlation in local factors. The earlier is rather interesting and shows some underlying dynamic worth further research.
5. Robustness checks

In order to examine robustness, the spillover framework was applied to data with a number of different parameter sets, on the whole time-span. Robustness will be evaluated through the total connectedness of the analysed time series. Results are shown on Table 16.

It is evident that the results are relatively robust to changes in both parameter of the Diebold-Yilmaz framework. Only by choosing the predictive horizon to 1 day would give substantially different results. That shows, that for one period forward, spillover dominates the forecast error variance, and the role of own variance shares is negligible. This is extremely true in the case of local factors. This may be due to the fact that volatility is forecastable 1-day forward relatively accurately – therefore virtually all the forecast error variance comes from spillover. This phenomenon – volatility clustering - is utilised by the ARCH-GARCH model family.

One can also observe, that total connectedness values are – ceteris paribus – declining as the order of the underlying VAR-model increases no matter the predictive horizon. On the other hand, total connectedness increases as the predictive horizon increases – apart from the sharp drop from 1 to 2. The explanation of these are rather difficult. The earlier means that including more lagged values of volatility makes a larger part of the forecast error variance attributable to shocks in the volatility of the time series. This is a surprising finding.

The latter means that a longer predictive horizon lowers the part attributable to shocks in the same time series. I find that this contradicts the earlier phenomena, and remains a puzzle. The results of robustness checks are strengthening rationale in the parameters chosen (2nd order VAR, with 10 days of predictive horizon).

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Table 15: Robustness checks on VAR lag and predictive horizon used. Table contains Total Spillover on the whole time-span.
Source: Bloomberg, own calculations.
6. Summary & concluding remarks

In my thesis, I took a diversification point of view and thoroughly analysed the volatility time series of 16 major global stock indices. Firstly, a state-space approach helped extract the global and 4 regional volatility factors from 16 globally relevant and geographically representative stock market indices’ volatility. This step was important as it allowed to distinguish volatility stemming from global, regional and local causes. The analysed four regions were North-America, South-America, Europe and Asia. From each region, 4 major stock indices were chosen.

A Global-Local, and a Global-Regional-Local setting was applied to data, from which the latter produced more sophisticated results, therefore the latter is used and presented here as well. The global factor is persistent with an autoregressive coefficient of 0.85. The regional factors are much less persistent, with their coefficients being in the range of 0.2-0.38. The individual volatilities’ betas on the global factor are around 0.17-0.34. The SHCOMP (China) and the IPSA (Chile) index has the lowest values, while S&P500 (USA) and FTSE100 (Great-Britain) has the largest. The extremely high persistence (0.79) of the idiosyncratic factor in terms of the Shanghai index further strengthens its independence from global and regional causes of volatility.

Subsequently, a variance decomposition analysis was performed in order to acquire information about the importance of global, regional and local factors of volatility. The results show that the global factor is a decisive factor in North-America, and Europe, and a somewhat less important in South-America and Asia. This result can be interpreted as the global factor is driven mainly by stock exchanges in the first two continents.

However, the importance of regional factors shows a much more mixed picture. Their relative share spread in the range of 2% and 38%, and it is relatively different between stock indices in the same region. This – along with spillover results may indicate regional leaders. The idiosyncratic, or local parts also show great variability. In the case of the S&P500, it’s share in variance is very close to zero, while SHCOMP’s volatility is driven by local factors mainly, it’s share being 66%. This is a good indicator of stock market volatility independence, which makes SHCOMP a valuable diversification option.

Then, the Diebold-Yilmaz Spillover Framework is applied to the individual volatilities and to the local factors, which are created by filtering out the global and regional factors’ effects from
Volatilities show high total connectedness with North-American indices, FTSE100 and MEXBOL being net transmitters, and others net receivers. The Nikkei225 index is the most affected by volatilities of other stock indices, which is a surprising finding concerning the status of Japan as “safe heaven”. This view is strongly contradicted by my analysis. Along with previous findings, my thesis strongly suggests that Chinese stocks form an outstanding diversification option given their independence from the rest of the world. Local factors exhibit lower total connectedness and the indices are very close to 0 in terms of net spillover.

The rolling window analysis firstly shows that total connectedness varies substantially in time, and that surprisingly local component can be just as much connected sometimes as the full volatility. It is also shown, that there is a strong connection between the value of the global factor, and the difference in volatility and local factor connectedness.

Secondly, regional net spillover Robustness checks show that the results are relatively stable against changing parameters of the methodology.

My thesis provided some insights about how the volatility of financial markets are connected, and also had a strong and extremely relevant finding. The Shanghai Composite Index seems like a good diversification tool as it moves totally independently from shocks in other stock indices volatility time series. The same cannot be said about Nikkei225 which is receiving a substantial amount of volatility from other stock indices.
7. References


