Fifty Shades of Risk

Demystifying Liquidity Risk Beyond High Frequency Trading in CDS Markets

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Chapter 3. Empirical results and Monte Carlo simulation

3.1. Empirical results

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Introduction

After the Flash Crash in May 6th, 2010, the financial markets had come to understand the importance of algorithmic trading and its drawbacks, especially when it comes to the so-called High Frequency Trading. This newly applied trading technique made investors to open or close their positions in nanoseconds which is even faster than a blink of an eye. Many economists, investment specialist and traders argue whether HFT is beneficial for the markets. Even more people question that HFT instead of providing liquidity, is simply just undermining the order book and causing breaks with immense losses. On the other hand some say HFT does pour money into the market via huge leverage, making the exchanges more efficient and flexible through narrow spreads, higher volumes and with more frequent capital turnover.

The reason why I have chosen this topic is because High Frequency Trading and its bold controversies is a recently discovered, but still grey and shady corner of the financial markets. Many theories were given birth by several financial or technological experts over the topic, although this rather new phenomenon in a perplexing combination with the latest developments still raises fear or doubt. Almost all of the theories are so emotion-packed, subjective or even biased that is really difficult to see the bigger picture or draw an objective and professional conclusion. Interestingly, there are two camps of opinion about High Frequency Trading: those who wholeheartedly advocate this tremendous implementation of technology, and there are people who are completely against this concept and would crucify these high-octane traders on a burning cross. The debate is fierce and the simile was intentionally exaggerating to emphasize how tough the discussion has become over the recent years when it comes to High Frequency Trading. Despite the overheated feelings, this trading procedure is nothing more or less than an evolution of an operating system that has been deployed for decades. Moreover due to the disgruntled investors and market participants who lost their money on exchange had fabricated such distortive misconceptions over High Frequency Trading that other “ordinary investors” started to demonize this technique as well. The most common misbeliefs are that flash orders are just a legal way of front-running; high frequency trading generates undesired noise into the markets; withdraws liquidity in times of crashes or in volatile trading days; provides phantom liquidity; High Frequency Trading has an unfair advantage by using sponsored access and/or co-location, and lastly not adding any benefits to the markets while pocketing formidably huge profits at the expense of these “ordinary investors”.
The main focus of my research is to examine market liquidity affiliated with High Frequency Trading through an objective point of view, considering many practical and theoretical aspects and tools accompanied with liquidity measures. Furthermore this study narrows its focus and will discuss market liquidity in Credit Derivative markets (specially, in Credit Default Swaps, CDS) because they share all the characteristics that would prevail if all misconceptions were proven to work (huge volatility, price discrepancies, mini crashes, escaping liquidity). According to the literature and a rare consensus, High Frequency Trading loves liquid markets like Equity or FX (less Futures/Forwards and even less Commodity) because it can close its positions superfast, it needs infinite supply and demand to fulfill its quotes. CDS markets are getting more liquid year by year, but still fragmented enough to be the perfect habitat for High Frequency predators, this will be discussed later in the study. Moreover, according to my in-depth research none of the authors have ever tried to examine the High Frequency Trading and its feasibility in Credit Derivative markets. Another reason I chose this market segment is because Credit Derivatives were one of the key factor causing the credit crunch in 2008, so this gives additional excitement to the already hot-topic. Testing and comparing results on Credit Derivative markets hopefully confirms the opposite of the common misbeliefs related to High Frequency Trading, because if those theories could not even hold in an extremely hazardous market, then a classic exchange would be conveniently safe and liquid.

In Chapter 1, I will discuss the most important characteristics and definitions related to High Frequency Trading with a short framing of market microstructure. Credit Default Swaps and major Credit Derivative indices will be under my microscope and lastly, the nature of a high frequency trader will be properly defined. Chapter 2 will give theoretical approach to liquidity and it will introduce the relevant literature of thorough methodologies in liquidity measures and analysis. Liquidity and main concepts will be dissected through a four dimensional analysis, including market tightness, -breadth, -depth and market resiliency. Implicit and explicit costs will be analyzed and will set assumptions for a proper Monte Carlo simulation. Chapter 3 presents the empirical results and the simulation. Last but not least, Chapter 4 gives the conclusion and mentions possible areas for further examinations.
Chapter 1. Definitions and theoretical approach

In this section, I will introduce the most essential concepts and expressions that may occur in the topic of High Frequency Trading. Consistently, the abbreviation for High Frequency Trading will be used hereafter as HFT.

1.1. The definition of High Frequency Trading

High Frequency Trading (HFT), is a mysterious and a complex expression to define, especially in circumstances of overheated debate. Many professionals who tried to define HFT are hardly ever approached the core meaning what it really is because some definitions are lacking enough technological consciousness or enough financial rigor. The task is compelling because a common definition of HFT yet to be found. The simplest and almost the best definition of HFT was created by Irene Aldridge: “High Frequency Trading is a form of automated trading that employs: algorithms for decision making, order initiation, generation, routing or execution for each individual transaction without human direction.” (Aldridge, 2013, pp 37-38). I think this is a fair approach but still, this definition is too technological and maybe even a bit confusing because it resembles to algorithmic trading or systematic trading which are completely different expressions (later on, I will define them as well). I think a good attempt to define HFT was made by Martin Wheatley, (CEO of Securities and Future Commission in Hong Kong and former deputy chief executive of the London Stock Exchange), because it comprises both technological and financial rigor. In the article in Financial Times Wheatley defined HFT as: “HFT is the execution of trading strategies based on computer programs or algorithms to capture opportunities that may be small or exist for a very short period of time.” (Wheatley, 2010)

Wheatley also identified three important characteristics of HFT: “(1) high volume of trades on a daily bases with low level of profits per trade; (2) extreme short stock holding period; (3) submitting numerous orders and (4) no significant positions overnight.” (Wheatley, 2010). In my opinion this definition covers the best what HFT is all about. Another great approach from Michael Durbin, “HFT is a way of buying or selling securities wherein success depends on how quickly you act, where a delay of a few thousandths of seconds, or milliseconds can mean the difference between profit and loss.” (Durbin, 2010, p.5.). A definition from Jonathan A. Brogaard is really similar to Durbin’s theory: “Subset of algorithmic trading where a large number of small-in-size orders are sent into the market at high speed, with round-trip execution
times usually measured in milliseconds.” (Brogaard, 2010, p.3) The first distinctive feature of HFT is terrific speed and short time frames for establishing and liquidating positions. The above mentioned definitions display consensus that HFT is a superfast type of trading. We are talking in milliseconds or even nanoseconds, a speed that is difficult to realize how fast really is. 1 second equals 1,000 milliseconds which equals 1,000,000 microseconds and that equals 1,000,000,000 nanoseconds. Even light travels 30 centimeter in 1 nanosecond. The most state-of-the-art robots trade in 740 nanoseconds… “The nearly two-minute sample of tick-data for SPY contained over 2,000 observations. 14:00:16:400 to 14:02:00:00 GMT on November 9, 2009. The number of quotes observed on November 9, 2009, for SPY alone would comprise over 160 years of daily open.” (Aldridge, 2013, p. 98). Another prominent characteristics of HFT is related to algorithmic trading. As Aldridge, Wheatley and Brogaard say, HFT is based on algorithms but it cannot explicitly be considered algorithmic trading (AT), HFT is rather a subset of AT. “High Frequency Trading developed in the 1990s in response to advances in computer technology and adoption of the new technology by the exchanges.” (Aldridge, 2013, p.32) Special care must be taken to distinguish HFT from algorithmic trading, electronic trading and systematic trading.

1.2. Basic terminology for algorithmic trading

 “Algorithmic trading is more complex than electronic trading and can refer to a variety of algorithms spanning order-execution processes as well as high-frequency portfolio allocation decisions.” (Aldridge, 2013, p.34) Electronic trading refers to the transmission of orders electronically. Since most orders nowadays are transmitted via computer systems, the expression for electronic trading is rapidly becoming old-fashioned. “Algorithmic execution makes decisions about the best way to route the order to the exchange, the best point in time to execute a submitted order if the order is not required to be executed immediately, and the best sequences of sizes in which the order should be optimally processed.” (Aldridge, 2013, p.34). Another great definition for AT is “Algorithmic trading is the use of computer algorithms to automatically make certain trading decisions, submit orders and manage those orders after submissions.” (Hendershott and Riordan, 2011, p.2). A great difference incur between HFT and AT that AT does not make portfolio allocation while it is the second core element for HFT. “Algorithmic trading does not usually make portfolio allocation decisions; the decision about when to buy or sell which securities are assumed to be exogenous.” (Aldridge, 2013, p.32). In spite I have mentioned above the Flash Crash, crashes had happened before the age of HFT,
when only rudimental robots traded day to day. For instance there is the 1987 market crash which was widely attributed to automated trading, then known as program trading (synonym for AT). HFT is a much more sophisticated system which comprises efficient strategies, pricing engines, stock-option-futures quoting generator, data feed listener and even a proper risk manager system. “True HFT systems make a full range of decisions, from identification of underpriced or overpriced securities through optimal portfolio allocation to best execution.” (Aldridge, 2013, p.35) Although the automation, in case of HFT, the stress is rather on the short position holding times which is one day or shorter in duration, usually with no positions held overnight. This means HFT works delta neutral overnight, ergo mitigates risk beyond market movements. The expression for short-term is often called “low-latency” which refers to “…trading that utilizes fast connectivity between traders and exchanges.” (Aldridge, 2013, p.67). It is designed to reduce response time by proximity and by co-location (also colo and see later in Chapter 2).

1.3. Quick review of market microstructure

Before I analyze the true nature of a HF trader, this subchapter is dedicated to market microstructure and trading mechanism in order to help to precisely understand my further arguments throughout this paper. According to a study from Bank of International Settlements (BIS) three factors determine market liquidity: (1) structure of the product, (2) market microstructure and (3) behavior of market participants. Sticking to the structure of BIS, firstly, market microstructure comes under my scope, secondly products will be determined in subchapters of credit default swaps and credit default swap indices. Only then the behavior of a HF trader will be analyzed in the last subchapter.

“Market microstructure examines the process of occurrence of investors’ latent demands in market prices and sizes. (Madhavan, 2000) There is another specific definition which says: “It is the study of the processes and outcomes of exchanging assets under a specific set of rules.” (O’Hara, 2008, p.1). We can distinguish two types of markets: limit order market (or auction market) and the dealer market1. “In limit order markets the final investors interact directly; their bids and asks2 are consolidated in a limit order book (LOB) according to price priority, so that higher bids and cheaper offers are more likely to be executed.” (Foucault, Pagano and Röell, 2013, p.16). Additionally, in order driven markets, since there is no intermediary, the liquidity is provided by pure demand and supply. In this case, “participants with limit orders are liquidity

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1 The former is often called as order driven and the latter is quote driven
2 Ask and offer are usually used as synonyms
suppliers (makers) and investors with market order are liquidity demanders (takers).” (Lyons, 2001)

As opposed to limit order markets, in dealer markets, final investors are obliged to trade with a specialized intermediaries\(^3\) at the bid and ask quotes submitted by these dealers or market makers. Furthermore, in dealer markets, the trading process follows an inverse mechanism: investors do not trade directly with each other. Unfolding the term of dealer markets, there are two distinctive segments which must be specified: the retail market and the wholesale\(^4\) segment. In the first type, dealers serve final investors and in the second one, “dealers trade with each other to share inventory risk.” (Foucault, Pagano and Röell, 2013, p.25) Inventory risk is the risk of a “potential loss when the value of his inventory declines in price due to natural market movements.” (Aldridge, 2013, p.258). For instance the market maker accumulates a long position in a downward trending market, or conversely, shorting when prices soar.

The interdealer markets are typically known for high volumes, according to Bank of International Settlements (BIS) “it accounts for about 80 percent of all foreign exchange market volume which includes pension funds, mutual funds, hedge funds, HFT and central banks or other sovereign wealth funds.” (Rime and Schrimpf, 2013)

Brokers differ from dealers in two aspects: first, they are counterparties between investors and they only execute orders while dealers are not counterparties and their activity falls far beyond just executing. Dealers also help investors to fill their quotes. Second, brokers do not take inventory risk but dealers do so. These facts also prove the evidence that high frequency traders are much more similar to market makers as it is described later. Brokers and dealers usually referred as the “sell side” and investors as “buy side” of a trade.

The next step to examine a complete market microstructure is to dissect the type of orders. Several can be distinguished, for instance good-till-cancel, good-till-day (cancel if not filled by the close of trading); immediate-or-cancel (try to fill at least some now and cancel the rest); fill-or-kill (cancel if all of them cannot be filled right now, or in a more precise financial term: cancel if the matching liquidity is not immediately available). Although these orders do exist, most market participants rely only the most conventional types of orders such as market order and limit orders, even HFT as well, even though according to Hautsch and Huang (2011),

\(^3\) Dealer or market maker are used for specialist as synonyms

\(^4\) Or interdealer market
95 percent of all limit orders are canceled within one minute from the time the orders are placed on Nasdaq.

A market order pays the best offer or receives the best bid. A market order is the most aggressive order because it crosses the spread. A market ask is matched with the best bid, and conversely a market bid is matched with the best ask on the opposite side of the LOB. When sells happen at the bid we are saying: “hitting the bid”. When buys happen at the offer it is called “lifting the offer”. “A market order consumes the best bid (ask) in its entirety, and then proceeds to be matched sequentially with next available best bid (ask) until the size of the market order is fulfilled.” (Aldridge, 2013, p. 93) A market order pays several costs and suffers from many risks including the spread, the incurring transaction fees and the price of uncertainty of course. Price of uncertainty can be the most hazardous component associated with market order because it has a high probability that from the time the order is submitted and to the time it is executed the market price may “slip”. It is often called as “cost of slippage” (more about costs in Chapter 2). On the other hand, market orders are perfect for immediate and guaranteed execution which is crucial for any HFT firm considering that liquidating positions in an extreme short period of time should be implementable.

A limit order specifies the price at which the investor is willing to trade. A limit buy order maximizes the price at which the trader is willing to buy on a given size. A limit sell order states the minimum price at which the seller will accept the deal. A limit buy order posted above the best bid becomes the new best bid, and a limit sell order posted below the best ask becomes the new best ask. If these do not happen, the limit order will be added into the LOB, where it sits until the market price reaches the order and a market order is filled against it. Limit orders ultimately tend to avoid of crossing the spread and thus limit orders have no incurring cost at all in interdealer markets or in limit order markets (no broker fees incur). Yet limit orders suffer from the risk of non-execution. The market may move against the investor causing immense losses. With a limit order an investor runs the risk that he cannot liquidate his position, escaping from the market will be impossible.

The dichotomy of reducing costs but also pursuing immediacy accompanies the entire history of HFT. For this reason, an optimal high HFT strategy commingles these two types of orders. Yet, there is a legitimate strategy to optimize cost between transaction time without using market orders: “To increase the chances of filling the entire order, therefore, the trader is likely to place a limit order with a larger size than his intended order, with the explicit hope that the
fraction of the order that will get filled is of the exact size as his intended order.” (Aldridge, 2013, p.78)

As price priority has mentioned before, the Intercontinental Commodity Exchange (ICE) has recently switched to pro-rata execution schedules. The previous quote matching principle was price/time priority execution. In summary, this principle had rewarded the fastest investor in the market because orders had been clustered by timestamps. This was one of the triggering cause of Flash Crash. Now the allocation is proportional to the size of each order book according to Eurex Exchange. The main advantage of the pro-rata matching is the built-in incentives for traders to place large limit orders, and, therefore to bring liquidity to the exchange.

1.4. Credit default swaps

“A credit default swap (CDS) is a contract used as insurance against credit event.” (Wilmott, 2009, p.472) Such event might be a default, a restructuring, a bankruptcy, or even a drop in the borrower’s credit rating. “One party pays interest to another for a prescribed time or until default of the underlying instrument. In the event of default the counterparty then pays the principal in return” (Wilmott, op. cit.) The seller of the CDS will pay off the buyer who makes periodic payments to the seller.

Anyone can purchase a CDS, even buyers who do not hold the loan instrument and who have no direct insurable interest in the loan. These naked credit default swaps allow traders to speculate on the credibility of reference entities. A CDS is usually linked to a “reference entity” or a “reference obligor”. Naked CDS are prohibited on European sovereign debt.

An investor or speculator may buy protection to hedge the default risk on a bond or other debt instrument if and only if the investor holds the reference entity’s debt, in this case a CDS can act as a hedge.

The settlements can be executed either in physical- or in cash settlement. A company with a higher CDS spread is considered more likely to default. Spread represents the fee of protection against the credit event from the seller’s point of view.

5 These are called „naked CDSs”
The investor selling the CDS is viewed as being “long” in the credit quality if and only if the investor owns the bond. The opposite holds, if the investor buys protection he is going “short” on the CDS and on the underlying credit quality.

1. **Figure** - *Revenue streams between counterparties*

![Diagram showing revenue streams between protection buyer and seller with no default and default scenarios.]

To outline how vast the CDS markets actually are, a table has been made to visualize the most essential pieces of information and characteristics of the credit derivative market.

1. **Table** – *Extent of credit derivative market ICE clear in America (Source: ICE web site)*

<table>
<thead>
<tr>
<th></th>
<th>Index</th>
<th>Corporate Single Names</th>
<th>Sovereign Single Names</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Instruments</strong></td>
<td>129</td>
<td>410</td>
<td>21</td>
</tr>
<tr>
<td><strong>Number of Trades Cleared</strong></td>
<td>989,109</td>
<td>650,768</td>
<td>93,03</td>
</tr>
<tr>
<td><strong>Gross Notional Cleared</strong></td>
<td>$45.7 trillion</td>
<td>$3.61 trillion</td>
<td>$933 billion</td>
</tr>
<tr>
<td><strong>Buy-side Notional Cleared</strong></td>
<td>$19.1 trillion</td>
<td>$63.9 billion</td>
<td>$17.1 billion</td>
</tr>
<tr>
<td><strong>Open Interest</strong></td>
<td>$475 billion</td>
<td>$325 billion</td>
<td>$84.2 billion</td>
</tr>
<tr>
<td><strong>Clearing Members</strong></td>
<td></td>
<td></td>
<td>30</td>
</tr>
</tbody>
</table>
## 2. Table - Extent of credit derivative market ICE clear in Europe (Source: ICE web site)

<table>
<thead>
<tr>
<th></th>
<th>Index</th>
<th>Corporate Single Names</th>
<th>Sovereign Single Names</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Instruments</strong></td>
<td>60</td>
<td>176</td>
<td>7</td>
</tr>
<tr>
<td><strong>Number of Trades Cleared</strong></td>
<td>465,678</td>
<td>608,492</td>
<td>15,001</td>
</tr>
<tr>
<td><strong>Gross Notional Cleared</strong></td>
<td>€16.4 trillion</td>
<td>€2.88 trillion</td>
<td>$305 billion</td>
</tr>
<tr>
<td><strong>Buy-side Notional Cleared</strong></td>
<td>€28.5 billion</td>
<td>€3.03 billion</td>
<td>$1.09 billion</td>
</tr>
<tr>
<td><strong>Open Interest</strong></td>
<td>€166 billion</td>
<td>€277 billion</td>
<td>$73.0 billion</td>
</tr>
<tr>
<td><strong>Clearing Members</strong></td>
<td></td>
<td></td>
<td>22</td>
</tr>
</tbody>
</table>

CDSs are one of the weapon of choice in arbitrage strategies called *Capital Structure Arbitrage*. This strategy builds on the fact that of a company’s stock price and its CDS spread should follow negative correlation. If the company’s performance increases, its share prices must rise and its CDS spread should tighten, because it is less likely to default on its debt in a short period of time. However, if the company faces a crisis, the CDS spread should widen and its stock price must plunge. The strategy exploits price discrepancies between company’s debt and equity. For example if “Bad Company” announced bad news about its last quarter performance and its share price has plummeted by 10%, but the CDS spread remained untouched, the investor might reasonably expect that the CDS spread will rise, hence he goes long on the CDS (buying a CDS) while simultaneously hedging himself by buying the underlying stock.

### 1.5. Credit default swap indices, CDX and iTraxx

“A credit default swap index is a credit derivative used to hedge credit risk or to take a position on a basket of credit entities.” (Wilmott, op. cit.) Unlike a CDS, which is a traditionally over-the-counter (OTC) credit derivative, a credit default swap index is completely standardized credit security and may therefore be more liquid. Just like equity index ETFs (for example SPY U.S. Equity, which is a basket of several securities) it can be cheaper to hedge a portfolio of credit default swaps with a CDS index than it would be to buy many single name
CDS to achieve a similar effect. Traders also use CDS indices to speculate on changes in credit quality. There are two major families of corporate CDS indices: CDX and iTraxx. Both owned by Markit Group Limited.

From October 2013, the U.S. legislation, specifically with the Dodd-Frank Act, CDS indices are required to be standardized and traded on electronic trading platform called Swaps Execution Facility (SEF) provided by CFTC which perfectly suitable for HFT systems. Indices are cleared through with ICE or with Chicago Mercantile Exchange (CME) as central counterparties (CCP).

Markit CDX indices are a family of tradeable CDS indices covering North America and Emerging Markets, while Markit iTraxx indices are a family of European, Asian and Emerging Market tradable credit default swap indices. 40% of total CDS notional is in DTCC. Main customers are banks, asset managers, hedge funds, ETF providers of which are quality counterparties. A wide range of products can be found, for instance CDX North American Investment Grade or North American Investment Grade High Volatility, North American High Yield, North American High Yield High Beta, Emerging Markets and Emerging Markets Diversified. This list of menu definitely suffices investors’ taste considering their risk-return appetite.

According to Markit Limited Group, the key benefits are:

- **Trading efficiency**: “Ability to trade large sizes quickly and confirm all trades electronically” (Markit)
- **Liquidity**: “Wide dealer and industry support, allowing for significant liquidity in all market conditions” (Markit)
- **Transparency**: “Pricing freely available daily on all indices, and all index characteristics standardized and documented” (Markit)
- **Data integrity**: “Contributed prices from multiple leading banks, with rigorous quality control process applied” (Markit)

ISDA Determinations Committee declares the credit event, in that case, the index is reversed. As credit event comprises failure to pay or bankruptcy filings and restructuring, obviously the protection seller makes a payoff to the protection buyer on the credit event settlement date (similarly to single name CDS). The payoff equals with the amount of payment of protection that would had been bought on a single name CDS.
These CDS indices individually possess a credit curve: set of credit spreads across range of time-to-maturity (usually upward sloping: risks of default increase over longer period of time, negative slope: serious short-term risks in default). The notion and logic resembles to term structure curves where returns or interest rates are figured in the same way representing the long and short-term risks and expectations.

1.6. The market role of a high frequency trader

This subchapter consolidates the ideas and facts that have been previously occurred in this study and tries to extrapolate these facts to a typical high frequency trader (hereafter HF trader).

As described above a HFT is an evolution of algorithmic trading and not a groundbreaking individual set of trading technique. Michael Durbin emphasize the fact that “the HF trader evolved from the ranks of the traditional market-maker, or specialist, whose primary source of profit was the spread between the prices at which he bought and sold.” (Durbin, 2010, p.6).

Moving on this line of idea, a HF trader is a selective market maker who uses superior execution speed and prediction capabilities to earn trading profit. Therefore a HF trader is essentially a hybrid of the market maker and short term predictor. A definition for the expression would also stand: a HF trader is a quantitative trader which uses statistical arbitrage. (pairs-trade or correlation arbitrage).

A HF trader also exploits the technological advances to achieve high speed on trading venues, including proximity and co-location. An ordinary HF trader hunts for micro price changes in a fraction of a second, therefore high leverage and capital turnover is used to profit on those price changes. A HF trader requires a highly liquid market for two reasons: (1) in order to liquidate his positions quickly (enough buying power is needed), (2) the transaction time of which the order consumes to be minimal. Furthermore, because a HF trader needs high execution speed he also need to quote close to the market price as automated market makers do so, since the HF trader wants his orders successfully and instantaneously matched. Hence, the HF trader is likely to stay on top of the book, submitting mainly market bids and asks. On the other hand, avoiding any incurring cost, a HF trader may rely only on limit orders, while an appropriate strategy exist to dissolve the trade-off between speed and cost. A participant with market order called liquidity taker, and a participant with limit order called liquidity maker, one
may conclude a HF trader taking away liquidity. Although it is not excluded, but a sufficient HF trader does both; making and taking liquidity.

Another hybrid characteristic of a HF trader is that works on a limit order market (no intermediary etc.) to avoid costs but in the meanwhile he operates in a way like market makers does in interdealer markets, trading directly with other market makers or dealers to share inventory risk. The interdealer market was outlined as a market with high volume because many institutional participant trades there as well, hence it is an excellent place to trade to a HF system. As I implied several times, a HF trader and a market maker are cognate species in the food chain, and since “a market-maker has no inherent interest in holding securities” (Durbin, 2010, p.32), a HF trader would also discard all his securities in a short time period. This is the reason why a HFT operates delta neutral overnight. Equally important that “automatization helps HF traders to manage inventory risk by, say, taking position in one market and hedging it almost instantaneously in another. Thus high frequency market makers may enhance market liquidity.” (Foucault, Pagano and Röell, 2013, p.40) Capital Structure Strategy might be such strategy. One may long or short in Equity or Bond market and hedge with CDX indices in parallel.
Chapter 2. Practical approach to market liquidity and modelling

In this chapter, I devoted an individual part for defining liquidity since it stands as the second major focus in this study and completely separated from the definitions in Chapter 1, which focused mainly on defining High Frequency Trading and took special care to understand its operation which determines market liquidity too. Subsequently, I present the most important models and methodologies to measure market liquidity and a thorough cost analysis is shown as it is also a determining factor when it comes to measuring liquidity. Furthermore, a second line of hypothesis will be posed throughout this chapter referring to market liquidity beyond high frequency trading in credit derivative markets.

“Liquidity is a degree to which an order can be executed within a short time frame at a price close to the security’s consensus value. Conversely, a price that deviates substantially from this consensus value indicates illiquidity.” (Foucault, Pagano and Röell, 2013, p.2) Viewing this theory, this not only proves why HFT desires liquid markets, but also proves the fact why a HFT will not take too much liquidity away from the books, because it would undermine its own operation. HFTs in illiquid markets are not able to exit rapidly, which is crucial for them (as I have suggested over the time). Later on, I will examine whether markets still remain liquid during HFT operation or not, then how volatility affects liquidity and vice versa.

According to a study of BIS “a liquid market is a market where participants can rapidly execute large-volume transactions with a small impact on prices” (BIS, 1999, p.5). As a consequence, one may conclude that HFT does making liquidity in the market, because its operation requires high volume and leverage. Therefore any HFT presence in a market eventually a contribution to liquidity.

Fisher Black deems an asset liquid in his article published in 1971 “if it can be sold in a short period of time, not too much below the price compared to one if the sell either way would happened on a longer time horizon” (Black, 1971). Black also framed a theory about the conditions of liquid markets that must prevail in any circumstances. It goes as follows: The bid-ask is always available for the investor, at which he can execute his transaction immediately. The bid-ask spread is always small and the investor, who not possess any insider information, is expected to execute a large volume transaction approximately at the best bid and best ask.
Another framework can be an underlying point to my further examination, a five dimensional analysis which dissects market immediacy, -tightness, -depth and market breadth. There is an additional metric of liquidity often referred as market resiliency, or Kyle’s lambda. Market resiliency is often a neglected and estranged son of the family of liquidity measures. Under my scope, market resiliency will enjoy a special role since its interpretation and application is suitable to measure liquidity under high frequency activity.

- **Tightness**: how far transaction costs diverge from mid-market prices. The smaller the transaction cost of a trade is, the more liquid the market is.
- **Depth**: the market’s ability to sustain relatively large market orders without impacting the price.
- **Breadth**: the width of bid-ask spread. The narrower it is, the more liquid the market is.
- **Immediacy**: the more instantaneous the execution is, the more liquid the market is.
- **Resiliency**: “the speed with which prices tend to converge towards the underlying liquidation value” or the “rate at which prices bounce back from an uninformative order flow shock.” (Kyle, 1985) This dimension is perfectly suitable to further examine liquidity under high frequency trading since HFT can produce imbalanced order flow and adverse price jumps. Tightness will be measured through cost analysis, the other three\(^6\) dimensions will be under my microscope via market liquidity models. Depth and resiliency will be discussed together as they are cognate measures.

Market tightness, depth and breadth are static dimensions of liquidity, while the market immediacy and market resiliency are dynamic dimensions

2.1. **Market tightness**

This subchapter gives a thorough cost analysis because costs are essential factors in market liquidity as they are able to be the barrier of entry. From every traders’ point of view, costs must be minimized (desirably to zero), to gain more profit from each trade. There is no such difference in high frequency trading especially because profits are razor thin.

2.1.1. **Transaction costs**

The development of computer technology over the last decades has led to a substantial drop in hardware and software prices. Consequently, technology-based algorithmic trading and

\(^6\) market immediacy will be left because this dimension was previously presented. The study has come to a conclusion that market immediacy prevails among exchanges since constant and high speed connectivity is available for every market participants, therefore no barrier obstructs liquidity reaching the markets.
high frequency trading have become cost efficient. Dozens of companies provide technology and other trading related services for example Perseus, Anova, Virtu Financial and Spread Networks. The most state-of-the-art technology service provided by these firms are wireless latency networks for instance hybrid laser metro connection, microwave transmitting, millimeter wave transmitting and so on. Perseus provides Ultra PrecisionSync™ Time Synchronization with PPS -1 nanosecond + (Precision Time Protocol) PTP 100 nanoseconds + (Network Time Protocol) NTP 250 nanoseconds. This global infrastructure makes possible a 24/7/365 constant connection to trading vendors with a 300 gigabit per second fiber optic backbone (optionally) between datacenters in Chicago, Pennsylvania and New Jersey at even a low cost. The microwave network costs between $13 and $26 million according to Perseus.

One must bear in mind that a HFT firm makes billions of dollars, $13 to $26 million dollar initial investment is relatively insignificant. In light of these facts, we must elucidate that investors leave millions of dollars either way at investment banks such as Citigroup for commissions executing the same trade with less profitability. According to Foucault, Pagano and Röell, it is a utopia and an idealized world where all participants are present on the market; submitting orders that reflect demand and supply of securities and a dealer carries out trades on a single equilibrium price that reflects a consensual fundamental value of a security. “Market players are not all simultaneously present on the market. Such continuous presence would be too costly in time, attention, and access costs.” (Foucault, Pagano and Röell, 2013, p.1). Well, the high frequency network services make possible a constant presence on exchanges and for relatively low price or even for free. This enhances the fostering of liquidity to reach markets at any time and at anywhere. This science-fiction technology advancements are unarguably pioneering in making nowadays trading more efficient and by vanishing the most remote memories of Kafkaesque broker driven markets when huge brokerage were accounted. As Irene Aldridge compares broker driven markets and low frequency trading to “the highly manual and therefore labor-intensive financial landscape of the 1970s was characterized by high transaction costs, leading to low turnover of securities; high degree of error associated with manual processing of orders, and relatively high risk of trading, as traders predominantly relied on their experience and intuition as opposed to science in making their bets on the markets.” (Aldridge, 2013, p.21). Transaction costs may have a wide range, but I tried to find the lowest exchange fees that occur in the market as a good benchmark to high frequency operating costs.
Table – NYSE Arca Rates
(Source: NYSE web site)

<table>
<thead>
<tr>
<th>Tier</th>
<th>Requirements</th>
<th>Rebate for Adding</th>
<th>Fees for Removing</th>
<th>Routing to NYSE</th>
<th>Routing to Other Venues</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tier 1</td>
<td>NYSE Arca Daily Adding as of % of US CADV in excess of 0.70%</td>
<td>$(0.0031) per share</td>
<td>$0.0030 per share</td>
<td>$0.0021/0.0023</td>
<td>$0.0030 per share</td>
</tr>
<tr>
<td>Tier 2</td>
<td>NYSE Arca Daily Adding as of % of US CADV in excess of 0.30%</td>
<td>$(0.0029) per share</td>
<td>$0.0030 per share</td>
<td>$0.0021/0.0023</td>
<td>$0.0030 per share</td>
</tr>
<tr>
<td>Tier 3</td>
<td>NYSE Arca Daily Adding as of % of US CADV in excess of 0.20%</td>
<td>$(0.0025) per share</td>
<td>$0.0030 per share</td>
<td>$0.0021/0.0023</td>
<td>$0.0030 per share</td>
</tr>
<tr>
<td>Step-Up Tier 1</td>
<td>NYSE Arca Daily Adding as of % of US CADV in excess of 0.15%</td>
<td>$(0.0295) per share</td>
<td>$0.0030 per share</td>
<td>$0.0021/0.0023</td>
<td>$0.0030 per share</td>
</tr>
</tbody>
</table>

Negative transaction costs known as rebates.

Table – U.S. Commission Rates – Per 100 Shares
(Source: Interactive Brokers)

<table>
<thead>
<tr>
<th>Broker</th>
<th>Commission</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interactive Brokers</td>
<td>$1.00</td>
</tr>
<tr>
<td>E-Trade</td>
<td>$7.99</td>
</tr>
<tr>
<td>Fidelity</td>
<td>$7.95</td>
</tr>
<tr>
<td>OptionsXpress</td>
<td>$8.95</td>
</tr>
<tr>
<td>Schwab</td>
<td>$8.95</td>
</tr>
<tr>
<td>TD Ameritrade</td>
<td>$9.99</td>
</tr>
<tr>
<td>Thinkorswim</td>
<td>$9.99</td>
</tr>
</tbody>
</table>

Normal exchanges charge traders for submitting market orders as taking liquidity away and offer rebates for submitting limit orders and making liquidity. On the other hand, inverted exchanges pay traders rebates for removing liquidity by submitting market orders and charge fees for placing limit orders. Boston Exchange is a typical example of inverted exchange.

Iceberg orders and other special “hidden orders” such as “push the elephant” or “reserve orders” are charged for higher broker commission fee, therefore why would a sensible HFT
resort to such costly strategy? In light the facts and sheer logic, a proper HFT tends to avoid any incurring cost of which likely to erode its profitability, thereby my view is narrowed only to market and limit orders through the rest of the study.

From the wrecks of Flash Crash that caused broken markets, a fair, transparent and cheap exchange has risen recently founded by CEO Brad Katsuyama. This is Investors Exchange (IEX). An exchange of which is dedicated to protect investors from high frequency predators and committed to transparency. Although IEX is a trailblazer among exchanges with the prudent and transparent philosophy of operation, but the biggest advantage lies not only on these factors, but on cost effectiveness. Its transaction costs are the lowest in the market, a table is presented about fees here

5. Table – Fees for matching and routing

<table>
<thead>
<tr>
<th>Standard Rate Adding Liquidity</th>
<th>$0.0009</th>
<th>Shares Priced $1.00 or Above</th>
<th>Away Venue Execution Cost to IEX plus $0.0001</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Rate Removing Liquidity</td>
<td>$0.0009</td>
<td>Shares Priced Below $1.00</td>
<td>Away Venue Execution Cost to IEX plus $0.0001</td>
</tr>
</tbody>
</table>

Moreover, IEX gives Alternative Trading Systems (ATS), Exchange membership, 10G physical port, logical port, physical connection and market data for free! On the flip side of the coin, IEX embedded to its operating system a 350 microseconds delay – IEX intentionally extends the time it takes for traders’ signal to reach its matching engine, thus negating any speed advantage, IEX does not allow any firm to put computers near a matching engine (co-location) and offers no rebates and a limited types of orders. One can conclude that HFT did improved market efficiency, but not for its own advantage. The merits are still accounted.

As a conclusion, markets are enjoying tight competitiveness by participants, thus reducing transaction costs and generating market transparency as a whole.

2.1.2. Taxes

“High-frequency trading generates short-term profits that are usually subject to the full tax rate, unlike investments of one year or more, which fall under the reduced-tax capital gains umbrella in most jurisdictions.” (Aldridge, 2013, p.129). For this reason, a HFT firm might use some tax evasion strategy to increase profitability.
2.1.3. Bid-ask spreads

Bid-ask spread is the cost of immediate liquidation of a position. The spread also can be viewed as the premium paid by investors demanding instantaneous execution through market orders. Bid-ask spreads are prone to increase during market uncertainty and instability as could be seen during the turbulent credit crisis in 2008. Widening spreads represent incipient illiquidity in markets, reducing probability of achieving profit. In a market where no one is willing to trade, a HFT would definitely escape regardless the prevalence of high spreads (as it can be seen as high premium). Consequently, liquidity related to short execution time of which plays high importance for any HFT rather than high spreads, and since volatility is the measure of uncertainty, then still why would a decent HFT has special interest in making markets volatile? Such notion is utterly non-sense even though Michal Lewis said in his book, “these firms are incentivized, for instance, to make the market as volatile as possible.” (Lewis, 2014, p.107) or “High Frequency traders benefited from volatile, fragmented markets.” (Lewis, op. cit.) Furthermore, if even any high price deviation occurred in the markets, that would provide opportunity of profitability, whereby more traders are willing to trade, then more liquidity is available and therefore the prevailing market turbulence would be shortly tamed by market participants by purely natural human behavior.

2.1.4. Slippage or latency costs

“Latency cost, commonly known as slippage, is the adverse change in the market price of the traded security that occurs from the time an investment decision is made until the time the trade is executed.” (Aldridge, 2013, p.130)

2.1.5. Opportunity costs

“The opportunity cost is the cost associated with inability to complete an order.” (Aldridge, 2013, p.134) Usually, opportunity cost is affiliated with limit order-based strategies, when the market price does not cross the specified limit price.

2.2. Market breadth

2.2.1. The quoted and normalized spread

\[ S = a - b \]

\[ s \equiv \frac{S}{m} = \frac{a - b}{m} \]

where \( m = \frac{a + b}{2} \)
Where $S$ is the quoted spread, $s$ the normalized spread, $m$ is the midprice, $a$ and $b$ are the asks and bids. The quoted spread for small trades is the most widely reported measure of illiquidity because “reflects the liquidity available at a given point in time for a hypothetical transaction.” (Foucault, Pagano and Röell, 2013, p.50)

2.2.2. The effective spread

Formally, the absolute effective spread is defined as:

$$S_e \equiv d(p - m) \quad \text{where } d = \begin{cases} 1 & \text{for buyer initiated} \\ -1 & \text{for seller initiated} \end{cases}$$

where $d$ is the direction of the market order triggering and $m$ is the midprice on the market prior to a transaction executed at price $p$. In relative terms:

$$s_e \equiv d\frac{p - m}{m}$$

“The effective spread can be seen as a measure of a transaction’s impact on the price, since it measures the deviation of the actual execution price from the midprice prevailing just before the transaction.” (Foucault, Pagano and Röell, 2013, p.51) This impact is previously called “slippage”. There are various methods to classify order direction for instance the Lee-Ready algorithm (1991) or Odders-White’s TORQ algorithm (2000) for higher accuracy. Lee-Ready’s notion is that the direction is buyer-initiated if its price is closer to the prevailing ask quote than to the bid, and seller-initiated if the converse stands. Any transaction priced exactly at the midprice is classified as a buy if the price is higher than the previous transaction price (“uptick”), a sell if lower (“downtick”). The algorithm classifies 85 percent of the transactions correctly, systematically misclassifies trades at the midpoint of the bid-ask spread. Despite the fact, I have implemented the Lee-Ready algorithm on my sampled data since TORQ algorithm proved to be too sophisticated and exceeds my knowledge to employ such method yet. Still, Lee-Ready does the trick.

2.2.3. The realized half spread

$$S_r = d_{\tau}(p_\tau - m_{\tau+\Delta}) = d_{\tau}(p_\tau - m_\tau) - d_{\tau}(m_{\tau+\Delta} - m_\tau)$$

This is a measure of the profit earned by the liquidity supplier on the transaction at time $\tau$. Substituting with the effective spread equation into the expression, this gives the average realized bid-ask spread:

$$\mathbb{E}(S_r) = \mathbb{E}(S_e) - \mathbb{E}(d_{\tau}(m_{\tau+\Delta} - m_\tau))$$
This expression above indicates that the average realized spread is smaller than the average effective spread if $\mathbb{E}(d_{\tau}(m_{\tau+\Delta} - m_{\tau})) > 0$, if the change in the midprice following a transaction is positively correlated with its direction. If the effective spread is small enough, liquidity providers would lose money on average, as $\mathbb{E}(S_p) < \mathbb{E}(d_{\tau}(m_{\tau+\Delta} - m_{\tau}))$. The choice of $\Delta$ is critical: it depends on how quickly market participants adjust their quotes after transaction. Under active and transparent market conditions, adjustment is generally fast, so a modest value of $\Delta$ is desired; too high value introduces unnecessary noise. Foucault advises $\Delta$ to be five or ten minutes.

2.3. Market depth and market resiliency

2.3.1. Kyle’s lambda

“The midprice tends to rise when buy orders arrive, to an extent that is positively correlated with their size. Symmetrically, it tends to fall in the wake of sell orders.” (Foucault, Pagano and Röell, 2013, p.56) If the midprice change is proportional to the buying or selling pressure the relationship can be expressed as follows:

$$\Delta m_{\tau} = \alpha + \lambda q_{\tau} + \varepsilon_{\tau}$$

where $\Delta m_{\tau}$ is the change in midprice over a fixed time interval, and $q_{\tau}$ is the order imbalance, that is, the total value of buy less less market orders executed in the same time interval. $\alpha, \varepsilon$ are the components of linear regression.

The reciprocal of $\lambda$ can be seen as a measure of market depth in that lower value of $\lambda$ means prices are less sensitive to order imbalance. To measure price impact $\lambda$ I have specified and run a model of regression. “While trading volume and order imbalance are certainly distinct concepts, they are likely to be correlated (days with larger order imbalances may well be the days with high trading volume). Therefore, one can estimate a regression of $|\Delta m_{\tau}|$ (the absolute value of price changes) on the trading volume $Vol_{\tau}$ (monetary value of total amount traded)” (Foucault, Pagano and Röell, 2013, p. 57) In this modified version, the slope can be interpreted as a measure of the price change associated with one additional unit of trading volume. Since Bloomberg do not provide sizes, only the aggregated volume, I resorted to this modified version of measuring Kyle’s lambda with total volume.
2.3.2. Amihud ratio and Amivest ratio

This measure is often related to the “illiquidity ratio proposed by Amihud (2002)

\[ y_\tau = \frac{|r_\tau|}{Vol_\tau} \]

where \(|r_\tau|\) is the absolute return for stock and \(Vol_\tau\) is the trading volume over a given period.

The inverse of Amihud ratio is the Amivest liquidity ratio. A low value of Amivest represents market illiquidity. Amihud is a good proxy for price impact as it highly correlated with high frequency measures of price impact.

2.3.3. Roll’s measure

Roll’s measure calculates bid-ask bounce, hence if it is low, we may conclude the market is liquid.

\[ S_{Roll} = 2\sqrt{-cov(\Delta p_{\tau+1}, \Delta p_\tau)} \]

Roll’s measure supposes that the security’s fundamental value, as captured by the midquote, follows a random walk:

\[ p_\tau = p_{\tau-1} + \varepsilon_\tau \quad \mathbb{E}(p_\tau - p_{\tau-1}) = 0 \]

where \(\varepsilon_\tau\) is mean-zero white noise (\(\mathbb{E}(\varepsilon_\tau) = 0\)) for all \(\tau\) which is reasonable for small time intervals and \(\mathbb{E}(\varepsilon_\tau \varepsilon_s) = 0\) for all \(t \neq s\).

We need several additional assumptions on the order arrival process, notably:

a. **Balanced order flow**: In reality, market buy and sell orders are not necessarily of equal probability, for example as I have implied, at the end of the trading day HFTs are closing all their positions. Unless the order flow is perfectly balanced (\(\eta = 1/2\)), Roll’s spread estimator is biased. Roll’s measure often underestimates the quoted effective spread.

b. **No autocorrelation in orders**: Roll’s method is also biased if there is a serial correlation in the trade direction \(d_\tau\). Suppose the direction of market orders is first order autocorrelated. The correlation between order directions in 15\(^{th}\) January was 0.23.

c. **No effect on midquote**: This is problematic because the direction of market orders does carry information.

d. **Constant (zero) expected return**: This assumption may fail because expected return can vary over time and serially correlated. In 15\(^{th}\) January, it was -0.0032%.
On the other hand, “Roll’s estimator works better the shorter the time interval over which the returns are measured, so that bid-ask bounce is larger relative to other determinants of returns” (Foucault, Pagano and Röell, p.64) This suggests that it is fairly suitable to my liquidity examination under high frequency trading’s time horizon. See how the assumptions work in the Appendix.

2.4. Setting assumptions for Monte Carlo simulation

My investigation would not be sufficient and thorough enough if Monte Carlo simulation (MC) had not been tested. Combining simulation with liquidity measures can add extra empirical results that leads to further inferences.

Assuming the price of the underlying security follows a Geometric Brownian Motion (GBM):

$$dS(t) = \mu S(t) dt + \sigma S(t) dW(t)$$

where $S(t)$ is the spot price of the security at time $t$, $\mu$ is the stationary increment $\mu = \alpha + \frac{1}{2} \sigma^2$ (where $\alpha$ is the annual drift\(^7\) and $\sigma$ is the annual volatility\(^8\)), $dt$ is the difference on a given time interval and $dW(t)$ is the increment of the Wiener process. $dW = \sqrt{dt}N(0,1)$. Wiener process is a stochastic process and the values of which are derived from a standard normally distributed set of random numbers.

Several assumptions come up considering GBM:

a. Finiteness
b. Continuity: the paths are continuous and differentiable everywhere
c. Markov property: distribution of returns are independent regardless the paths before.\(^9\) d. Martingale: unbiased random walk
e. Quadratic variation: $\sum_j^t (W_{t_j} - W_{t_{j-1}})^2 \to t$
f. Normality: it has mean zero and dt variance.

\(^7\) drift equals the average logreturns of which is annualized by multiplying with 250.
\(^8\) volatility equals the standard deviation of logreturns of which is annualized by multiplying with square root of 250
\(^9\) memoryless process property
Chapter 3. Empirical results and Monte Carlo simulation

In this chapter, I will evaluate the wide range of liquidity measures and methodologies which were previously presented. My implementation of extensive analysis uses real world data. The quality and the amount of data is crucial therefore I have downloaded the requiring data from Bloomberg using the *Historical Intraday-Bar* function via Excel add-in. The time interval was set to five minutes, taking Foucault’s advice when it comes to liquidity analysis.

3.1. Empirical results

3.1.1. Liquidity in SPY US Equity ETF

2. Figure – *SPY Price/Volume and Quoted spread*

Firstly, I examined the market liquidity in the SPY US Equity ETF (as a good benchmark to CDS markets later on) which gained its popularity from its high liquidity and often the main underlying asset for HFT. According to SEC reports, a purported HFT operation could have been observed in 24\textsuperscript{th} August, 2015 and in 17\textsuperscript{th} September, 2015. The entire study was built on two notions: (1) whether the hypothesis of high liquidity holds even under HFT operation; (2) whether the conditions stand in credit derivative markets as well.
The price/volume chart in 24th indicates two distinctive features: (1) the typical U shape liquidity pattern in markets which amplifies the fact that HFT robots open positions in the morning and close all of them before overnight (go delta neutral); (2) there was no significant drop in the prices with high volume capital turnover during the days. This hypothesis is underpinned by the second chart of quoted spreads which is the measure of illiquidity. The huge spikes occurred only on overnight and since we already know that HFT does not operate overnight, thus any price deviation from high volume could not be affiliated with HFT. The price/volume chart in 17th is much more informative: HFT did cause a steep fall in prices in the afternoon, yet the quoted spread did not widen severely.

The effective spread is the measure of a transaction’s impact on the price. The effective spread, on average, was 0.2159% in 24th August. This also indicates that the deviation (“slippage”) of the actual price from the midquote is relatively low. In 17th September, this was 0.0027% which is markedly lower even though an adverse price change occurred. This suggests that HFT does not contribute to large price impacts when it quotes with large volume.

The realized half spread is the measure of the profit earned by the liquidity supplier on the transaction. There are two ways of calculating the realized half spread: (1) one can take the simple average of $S_r = d_r(p_t - m_{t+\Delta})$ to get the expected value of this measure; (2) or just subtract $\mathbb{E}(d_r(m_{t+\Delta} - m_t))$ from the effective spread using the equation of $\mathbb{E}(S_r) = \mathbb{E}(S_e) - \mathbb{E}(d_r(m_{t+\Delta} - m_t))$. In 24th August, the realized half spread was 41.7841% by the subtraction, and 41.3840% by calculating the expected value of the half spread. Both are a great proxy, but the main conclusion is: the liquidity supplier did earn profit markedly under HFT operation because $\mathbb{E}(S_e) > \mathbb{E}(d_r(m_{t+\Delta} - m_t))$ therefore HFT cannot be blamed for ripping off ordinary investors and liquidity makers. According to subchapter 2.2.3., thus the midprice, following a transaction, is negatively correlated with its direction because the realized half spread is greater than the effective spread. In 17th September, the realized half spread was 1.0812% which is not as high as in 24th August, but liquidity makers still earned trading profit from providing liquidity considering HFT did cause severe break in prices. The same holds in this case as well: midprice and its direction is negatively correlated.
The regression model for 24th August, 2015 gives the following outputs:

3. Figure – Regression on SPY US Equity in 24th August, 2015

<table>
<thead>
<tr>
<th></th>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.15539935</td>
<td>0.073047268</td>
<td>2.127381</td>
<td>0.034409</td>
</tr>
<tr>
<td>X Variable 1</td>
<td>-0.00004179</td>
<td>0.000014521</td>
<td>-2.87791</td>
<td>0.004364</td>
</tr>
</tbody>
</table>

Therefore the model would be specified: \( \Delta m_\tau = 0.1554 - 0.00004179Vol_\tau \) where -0.00004179 (explanatory variable) represents Kyle’s lambda. The model would be interpreted as follows: a unit change in volume at time \( \tau \) would cause -0.00004179 change in the change of midquotes (response variable) at time \( \tau \) ceteris paribus. This low value of \( \lambda \) means prices are less sensitive to order imbalance and to large directional trades. Thus, high liquidity is available at time \( \tau \) and the market is resilient. Although, the intercept, \( \alpha \) would be desirable to be zero, but still it is relatively close to it, thus disregarding will not bias. The market is also deep, taking the reciprocal of the absolute value of \( \lambda \). The market depth was 23,929 in 24th August, 2015. The Amihud ratio, on average, \( \gamma \) was 0.00000003731 in 24th August, which represents low price impact on unit change of total volume at time \( \tau \), therefore the market is liquid. The Amivest ratio was 1,582,762,125.

The Roll’s measure has been proven not to work on the sampled data in 24th August, because the covariance of price changes was positive, thus the square root of a positive number multiplied by a negative one could not be taken. On the other hand, Roll’s measure was 0.1227 in 17th September, 2015 since the covariance between price changes was -0.0038. This indicates that the bid-ask spread bounced little under HFT stress, therefore suggests, market is liquid.
The regression model for 17th September, 2015 gives the following outputs:

4. **Figure** – *Regression of SPY US Equity in 17th September, 2015*

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.01370639</td>
<td>0.014544594</td>
<td>0.94237</td>
</tr>
<tr>
<td>X Variable</td>
<td>-0.000000287</td>
<td>3.97366E-06</td>
<td>-0.721655</td>
</tr>
</tbody>
</table>

Therefore the model is specified: \( \Delta m_\tau = 0.0137 - 0.000000287V_\tau \) where -0.000000287 (explanatory variable) is the Kyle’s lambda. A unit change in volume at time \( \tau \) would cause -0.000000287 change in the change of midquotes (response variable) at time \( \tau \) *ceteris paribus*. Prices were markedly less sensitive to order imbalance in 17th September than in 24th August even though the presence of HFT was much more disturbing in 17th September. Thus, high liquidity is available at time \( \tau \) and the market is resilient in 17th September as well. The intercept, \( \alpha \) is also closer to the desired value of zero. The market depth was 348,722, so the market is even deeper than 24th August. The Amihud ratio, on average, \( \gamma \) was 0.0000000024 and the Amivest ratio was 1,432,528,345. The market is liquid in 17th September too.

One might be curious about whether price changes are independent from each other. For this reason an additional figure shows the autocorrelation between prices. The regression line is relatively horizontal with a low value of \( R^2 \) (0.0005 and 0.0077), thus we can infer the price changes do not explain well each other, therefore they are independent. The conclusion is, no such domino effect occurred even in the of HFT stress in 24th August and 17th September as opposed to in 6th May, 2010.

5. **Figure** – *Autocorrelation between price changes for SPY US Equity*
Finally, CDX index ETF will be under my scope in this subchapter. This can be found in Bloomberg under the name of “5Y Markit CDX North America Investment Grade Index” (CDX.NA.IG) The index was issued in October, well after that mini flash crashes happened in 24th August or in 17th September, so I had to figure out when a presumable HFT operation occurred. The downloaded data covered the interval from 4th January 2016 to 10th March 2016. I was looking for steep price falls (almost vertical) with large volume and on a really short time interval. Only 15th January satisfied my assumptions, therefore I will examine the underlying data from 13th January to 19th January. (See in Appendix)

The chart of CDX price/volume shares all the characteristics with SPY US Equity ETF, particularly the overnight properties. Typical U shape pattern in daily volume, adverse price changes only in overnight market and the quoted spread remain relatively stable during trading day. The quoted spread is extremely close to zero (0.0100) which represents high liquidity in the credit derivative market and no evidence of HFT disturbance.¹⁰

The effective spread, on average, was 0.0054% in 15th January. This indicates that the deviation (“slippage”) of the actual price from the midquote is relatively low and quite identical compared to the price impact on SPY. **HFT does not contribute to large price impacts when it quotes with large volume in credit derivative markets as well.**

The realized half spread was -0.0398% in 15th January which indicates that a liquidity supplier would be not profitable under HFT turbulence in credit derivative market as opposed

---
¹⁰ Sometimes the quoted spread reaches the absolute zero
to SPY or other highly liquid security in the equity market. Yet, this value is still low to blame HFT causing breaks in credit derivative markets.

The Amihud ratio, on average, $\gamma$ was 0.0000000040 and the Amivest ratio was 252,800,463 which is **liquid but not as much as SPY ETF amid HFT stress.**

Roll’s measure was 0.0265 in 15th January, 2016 since the covariance between price changes was -0.00018. This indicates that the bid-ask spread bounced less then SPY did under HFT stress, therefore suggests, the credit derivative market is liquid.

The regression line is relatively horizontal with a downward slope. The $R^2$ (0.003) is low, thus we can infer the price changes do not explain well each other, therefore they are independent. The conclusion is, (like in the case of SPY) no such domino effect occurred in 15th January 2016 as well. (See Appendix)

The regression model for 15th January, 2016 gives the following outputs:

7. **Figure** – *Regression on 5Y CDX.NA.IG in 15th January, 2016*

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.00825912</td>
<td>-1.547121</td>
<td>0.12319695</td>
</tr>
<tr>
<td>X Variable 1</td>
<td>0.00000205</td>
<td>1.736616</td>
<td>0.08378237</td>
</tr>
</tbody>
</table>

**ANOVA**

<table>
<thead>
<tr>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Significance F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>1</td>
<td>0.01065389</td>
<td>0.010654</td>
<td>3.015834963</td>
</tr>
<tr>
<td>Residual</td>
<td>232</td>
<td>0.819574849</td>
<td>0.003533</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>233</td>
<td>0.830228739</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Therefore the model would be specified $\Delta m_\tau = -0.0083 - 0.00000205 Vol_\tau$ where -0.00000205 (explanatory variable) is the Kyle’s lambda. A unit change in volume at time $\tau$ would cause -0.00000205 change in the change of midquotes (response variable) at time $\tau$ *ceteris paribus*. Prices are not sensitive to order imbalance in 15th January. Thus, high liquidity is available at time $\tau$ and the credit derivative market is resilient as well. The intercept, $\alpha$ is also close to the desired value of zero.

To give further evidence of market resiliency and depth about credit derivative markets, a Monte Carlo simulation was implemented based upon the assumptions denoted in subchapter 2.4. The most critical condition is normality. I have employed a Chi-squared statistics ($\chi^2$ test) with a contingency table to examine the distribution of logreturns in 15th January.
Firstly, I have clustered the logreturns into twelve bins, then executed a histogram analysis through Excel Data Analysis tool-pack. That gave the observed frequency of logreturns. Then these frequencies were normalized by subtracting the average and divided by the standard deviation of logreturns. We get the value $z$. Then I have taken the standard normal distribution of these values. We get the value $\Phi$. Subsequently, a decumulation followed, calculating each probability of observed frequency. Then each probability were multiplied by the total number of observed frequencies, $n$. Then the following equation was used: 
$$
\chi^2 = \sum_{i=1}^{k} \frac{(f_i - nP_i)^2}{nP_i}.
$$
The next table sums up the main values. The null hypothesis is: the data set is normally distributed and the alternative hypothesis is that it is not normally distributed. The degree of freedom was calculated by $v = k - b - 1$ where, $k$ is the number of bins, $b$ is the number of estimated parameters. (in this case, there were two parameters: the average and the standard deviation). Thus, 12-2-1 equals 9. The p-value indicates the significance level at which the hypothesis is still not rejected. The critical Chi-squared value was calculated by $c_{right\,\,tailed} = \chi^2_{1-a}(v)$. Since my Chi-squared statistics is just below the value of critical Chi-squared point, we could not reject the null hypothesis, therefore the logreturns are truly normally distributed. Because I have proven that logreturns follow normal distribution in such a short period of time, a Geometric Brownian Motion (GBM) is a valid simulation technique combined with Monte Carlo simulation. Additional assumptions were proven as well, like martingale property. I have examined that the expected value of price changes is zero (well, almost zero, -0.002, but not as biased to reject the entire method). I have proven the Markov property too, the logreturns are independent, previous paths do not influence the next path. In the next subchapter, I will present the result of Monte Carlo simulation executed on CDX logreturns. The simulation was written in Excel Visual Basic Application, VBA. (see Appendix)
3.2. Monte Carlo simulation

In this subchapter, I have implemented a Monte Carlo simulation (based upon the assumptions I have denoted previously) to explore more statistical results about market resiliency and market depth in credit derivative markets. The method used the GBM formula for bid-ask and last prices. I also simulated volumes based upon its distributional properties. The simulation was run for 100 times and the calculation was onerous, the program runs for more than one hour…

The average value of lambda is 0.0000000007 and the average alpha is 0.00042 which is extremely close to the desired value of zero. Therefore a simplified and aggregated regression model would be described as follows: \( \Delta m_t = 0.00042 + 0.0000000007 Vol_t \). A unit change in volume would cause a 0.0000000007 change in the change of midquote ceteris paribus. Thus, the credit derivative market is resilient and deep in a simulated environment as well.

10. Figure - Results of Monte Carlo simulation
Chapter 4. Conclusion

The paper studied market liquidity under High Frequency Trading operation. The main hypothesis was proved that markets remain stable and liquid even during HFT stress. Both equity and credit derivative markets share all these characteristics. The quoted spreads remain narrow, only overnight opens wide. The effective spread have measured price slippage which is relatively low, both in equity and credit derivative markets, it hardly ever reaches 0.5%. The realized half spread indicated the profit earned by a liquidity supplier and one could examine that it was markedly high, even amid HFT turbulence, like in 24th August 2015. Therefore, providing liquidity is still profitable in equity market even though HFT did cause severe price breaks. On the other hand, credit derivative market still less profitable during HFT operation. Investor could lose money in credit derivative markets when he meets with a HFT robot. Despite the fact, both markets are resilient and deep, prices are less sensitive to order imbalance and to large directional trades and markets can sustain relatively large market orders without impacting the price. HFT does not contribute to large price impacts when it quotes large volume in credit derivative market as well.

HFT operation over the years contributed to a much more efficient and transparent markets. Costs are broken down by HFT systems, exchanges and several HFT firms provide cheap access to trading venues and even incentivize liquidity provision by offering rebates. A decent HFT system has no inherent interest to make markets volatile, fragmented and illiquid because it would undermine its only source of profit. Liquidity measures worked fairly under Monte Carlo simulation as well. The results were a great proxy to reality since most values “sitting in” near to the average and as going farther, the probability of an extreme value occurs declines. The study also gives evidence that price changes do not explain each other, therefore they are independent. On the other hand, there are several questions remained unanswered and an in-depth outlook would be desirable. For instance one might be curious about efficient market hypothesis (EMH) under HFT operation or market regulation due to HFT and maybe behavioral finance application could be revolutionized by HFT since they are robots… The psycho of HFT would be definitely an interesting topic to examine. Unfortunately they could have not been discussed with respect to the extent of this study but a Bachelor thesis might include these topics as well. Concerns and misconceptions about liquidity affiliated with HFT are successfully allayed in this paper, although HFT is still a grey and shady corner of the financial markets.
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Appendix

11. Figure - Finding presumable HFT operation

![CDX Price/Volume 01.04.-03.10.](image)

Excel Visual Basic Application, VBA script for Monte Carlo simulation

```
Sub regression()
    Range("AG21") = "Program started"
    Range("AH21") = Time

    Dim oszlop As Integer, variable As Integer
    oszlop = 41
    variable = 0

    Application.ScreenUpdating = False

    'Monte Carlo simulation starts here
    Do
        Range("AG27:AO44").Select
        Selection.ClearContents
        Range("AG17").Select
```
Dim average As Double, stdev As Double, S0 As Double, dt As Double, mu As Double
Dim SimNum As Integer, hossz As Integer

'LAST_PRICE
average = Range("AH10")
stdev = Range("AH11")
S0 = Range("AH12")
dt = Range("AH13")
mu = Range("AH14")
SimNum = Range("AH15")
hossz = 3667
Dim tomb() As Double
ReDim tomb(hossz)

'Generating random variables
For i = 1 To hossz
    veletlen = Rnd()
    random = Application.WorksheetFunction.Norm_S_Inv(veletlen)
    tomb(i) = random
Next i

For j = 1 To hossz
    Cells(j + 2, 11) = tomb(j)
Next j

'increments of the Wiener process
For i = 1 To hossz
    Cells(i + 2, 12) = Cells(i + 2, 11) * Sqr(dt)
Next i

'dY(t)
For i = 1 To hossz - 1
    Cells(i + 3, 13) = average * dt + Cells(i + 3, 12) * stdev
Next i
'Y(t)
Cells(3, 16) = 0
For i = 1 To hossz - 1
Cells(i + 3, 16) = Cells(i + 3, 13) + Cells(i + 2, 16)
Next i

'S-ABM
Cells(3, 19) = S0
For i = 1 To hossz - 1
Cells(i + 3, 19) = Cells(i + 2, 19) * Exp(Cells(i + 3, 16))
Next i

'dS and S-GBM
Cells(3, 25) = S0
For i = 1 To hossz - 1
Cells(i + 3, 22) = mu * Cells(i + 2, 25) * dt + stdev * Cells(i + 2, 25) * Cells(i + 3, 12)
Cells(i + 3, 25) = Cells(i + 3, 22) + Cells(i + 2, 25)
Next i

'--------------------------------------------------------------------------

'BID
Dim average2 As Double, stdev2 As Double, S02 As Double, dt2 As Double, mu2 As Double

average2 = Range("AI10")
stdev2 = Range("AI11")
S02 = Range("AI12")
dt2 = Range("AI13")
mu2 = Range("AI14")

'dbid(t)
For i = 1 To hossz - 1
Cells(i + 3, 14) = average2 * dt2 + Cells(i + 3, 12) * stdev2
Next i

'bid(t)
Cells(3, 17) = 0
For i = 1 To hossz - 1
Cells(i + 3, 17) = Cells(i + 3, 14) + Cells(i + 2, 17)
Next i

'bid-ABM
Cells(3, 20) = S02
For i = 1 To hossz - 1
Cells(i + 3, 20) = Cells(i + 2, 20) * Exp(Cells(i + 3, 17))
Next i

'dbid and bid-GBM
Cells(3, 26) = S02
For i = 1 To hossz - 1
Cells(i + 3, 23) = mu2 * Cells(i + 2, 26) * dt2 + stdev2 * Cells(i + 2, 26)
* Cells(i + 3, 12)
Cells(i + 3, 26) = Cells(i + 3, 23) + Cells(i + 2, 26)
Next i

' -----------------------------------------------
'ASK
Dim average3 As Double, stdev3 As Double, S03 As Double, dt3 As Double, mu3 As Double
average3 = Range("AJ10")
stdev3 = Range("AJ11")
S03 = Range("AJ12")
dt3 = Range("AJ13")
mu3 = Range("AJ14")

'dask(t)
For i = 1 To hossz - 1
Cells(i + 3, 15) = average3 * dt3 + Cells(i + 3, 12) * stdev3
Next i

'ask(t)
Cells(3, 18) = 0
For i = 1 To hossz - 1
Cells(i + 3, 18) = Cells(i + 3, 15) + Cells(i + 2, 18)
Next i
'ask-ABM
Cells(3, 21) = S03
For i = 1 To hossz - 1
Cells(i + 3, 21) = Cells(i + 2, 21) * Exp(Cells(i + 3, 18))
Next i

'dask and ask-GBM
Cells(3, 27) = S03
For i = 1 To hossz - 1
Cells(i + 3, 24) = mu3 * Cells(i + 2, 27) * dt3 + stdev3 * Cells(i + 2, 27)
* Cells(i + 3, 12)
Cells(i + 3, 27) = Cells(i + 3, 24) + Cells(i + 2, 27)
Next i

'Running regression
Application.Run "ATPVBAEN.XLAM!Regress", ActiveSheet.Range("$AE$4:$AE$3669")
    , ActiveSheet.Range("$AC$4:$AC$3669"); False, False, ,
ActiveSheet.Range("$AG$27"); False, False, False, False, , False

'Writing out lambda and alpha
Cells(oszlop + 5, 33) = Cells(44, 34)
Cells(oszlop + 5, 34) = Cells(43, 34)

'Writing out prices
Dim tomb2() As Double
ReDim tomb2(hossz)
For i = 1 To hossz
tomb2(i) = Cells(2 + i, 25)
Next i

For j = 1 To hossz
Cells(j + 3, variable + 40) = tomb2(j)
Next j
oszlop = oszlop + 1
variable = variable + 1

'Monte Carlo simulation ends here
Loop Until variable = SimNum

Application.ScreenUpdating = True

Range("AG22") = "Program finished"
Range("AH22") = Time
Range("AG23") = "Time elapsed"
Range("AH23") = Range("AH22") - Range("AH21")

MsgBox "Ready"
End Sub